Unit 6 Probability

Day 1 Counting Techniques

Warm Up

Homework Discussion

Homework Tonight

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Unit 6 Probability

Day 1
Fundamental Counting Principle
Other Counting Techniques

Probability

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I. **Introduction**

Probability Defined:

What do you know about probability?

Probability

Notes p. 1

I. **Introduction**

Probability Defined:

- General: Probability is the likelihood of something happening

- Mathematical expression:

$$\text{Probability} = \frac{\text{Number of desired outcomes}}{\text{Number of total outcomes}}$$

Today, we'll focus on counting techniques to help determine this total #!

II. **Basic Counting Methods for Determining the Number of Possible Outcomes**

a. **Tree Diagrams:**

Example #1: LG will manufacture 5 different cellular phones: Ally, Extravert, Intuition, Cosmos and Optimus. Each phone comes in two different colors: Black or Red. Make a tree diagram representing the different products.

How many different products can the company display?

b. In general:

- If there are *m* ways to make a first selection and *n* ways to make a second selection, then there are *m times n* ways to make the two selections simultaneously. This is called the **Fundamental Counting Principle.**

- Ex #1 above: 5 different cell phones in 2 different colors. How many different products?

$$5 \cdot 2 = 10$$

Practice

Ex #2: Elizabeth is going to completely refurbish her car. She can choose from 4 exterior colors: white, red, blue and black. She can choose from two interior colors: black and tan. She can choose from two sets of rims: chrome and alloy. How many different ways can Elizabeth remake her car? Make a tree diagram and use the Counting Principle.

$$4 \cdot 2 \cdot 2 = 16$$

Ex #3: Passwords for employees at a company in Raleigh NC are 8 digits long and must be numerical (numbers only). How many passwords are possible? (Passwords cannot begin with 0)

$$9 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 90{,}000{,}000$$

B. Permutations—Another way to "count" possibilities

a. Two characteristics:

1. Order IS important

2. No item is used more than once

Example #1

There are six “permutations”, or arrangements, of the numbers 1, 2 and 3.

What are they?

123	132
213	231
312	321

Example #2

How many ways can 10 cars park in 6 spaces? The other four will have to wait for a parking spot. ☺

(Use the Fundamental Counting Principle)

$$10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = 151200$$

b. Formula:

If we have a large number of items to choose from, the fundamental counting principle would be inefficient. Therefore, a formula would be useful.

First we need to look at **"factorials".**

Notation: n! stands for n factorial

Definition of n factorial:

For any integer n>0,

$$n! = n(n-1)(n-2)(n-3)\ldots(3)(2)(1)$$

Supplemental Example:

4! = 4•3•2•1

If n=0, 0! =1

Example #2 (revisited):

We could rewrite the computation in our example as follows:

$$10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1} = \frac{10!}{4!}$$

Furthermore, notice that $= \frac{10!}{4!} = \frac{10!}{(10-6)!}$

So, the number of permutations (or arrangements)

of 10 cars taken 6 at a time is 151200.

Generally, the ***Number of Permutations*** of *n* items taken *r* at a time,

$$_nP_r = \frac{n!}{(n-r)!}$$

How to do on the calculator:

n MATH PRB nPr r

Note: You'll have to know how to calculate these by hand, BUT remember you can check your work with the calculator!

c. EXAMPLE #3

In a scrabble game, Jane picked the letters A,D,F,V, E and I. How many permutations (or arrangements) of 4 letters are possible?

$$\frac{6!}{(6-4)!} = 360$$

Let's do both ways – by hand with the formula and in the calculator!

Practice Problems

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

1. Evaluate: (*By hand then using ${}_nP_r$ function on the calculator to check your answer*.)

   a. ${}_{10}P_3$

   720

   b. ${}_9P_5$

   15120

2. How many ways can runners in the 100 meter dash finish 1st (Gold Medal), 2nd (Silver) and 3rd (Bronze Medal) from 8 runners in the final? NOTE: This is a permutation because the people are finishing in a position. ORDER matters!

   336

C. Combinations

a. Two characteristics:
   1. Order DOES NOT matter
   2. No item is used more than once

Supplemental Example: How many "combinations" of the numbers 1, 2 and 3 are possible?

There is just 1 combination of 1, 2, 3 because order doesn't matter so 123 is considered the same as 321, 213, etc.

EXAMPLE:

While creating a playlist on your ipod you can choose 4 songs from an album of 6 songs. If you can choose a given song only once, how many different combinations are possible? (List all the possibilities)

We'll let A, B, C, D, E, and F represent the songs.

ABCD ABCE ABCF ACDE ACDF ADEF
ABDE ABDF ACEF
ABEF

BCDE BCDF BDEF CDEF
BCEF

There are 15 combinations!

b. Formula:

Making a list to determine the number of combinations can be time consuming. Like permutations, there is a general formula for finding the number of possible combinations.

Number of Combinations of *n* items taken *r* items at a time is

$$ {}_nC_r = \frac{n!}{(n-r)! \cdot r!} $$

How to do on the calculator:

n MATH PRB nCr r

Let's look at the Playlist EXAMPLE again

While creating a playlist on your I pod you can choose 4 songs from an album of 6 songs. If you can choose a given song only once, how many different combinations are possible? (List all the possibilities)

Let's do both ways – by hand with the formula and in the calculator!

Practice Problems

$$ {}_nC_r = \frac{n!}{(n-r)! \cdot r!} $$

1. Evaluate:

   a. ${}_4C_2$ b. ${}_7C_3$ c. ${}_8C_8$

2. A local restaurant is offering a 3 item lunch special. If you can **choose 3 or fewer items** from a total of 7 choices, how many possible combinations can you select?

3. A hockey team consists of ten offensive players, seven defensive players, and three goaltenders. In how many ways can the coach select a starting line up of three offensive players, two defensive players, and one goaltender?

Practice Problems

$$ {}_nC_r = \frac{n!}{(n-r)! \cdot r!} $$

1. Evaluate:

a. ${}_4C_2$

6

b. ${}_7C_3$

35

c. ${}_8C_8$

1

Practice Problems

$$ {}_nC_r = \frac{n!}{(n-r)! \cdot r!} $$

2. A local restaurant is offering a 3 item lunch special. If you can **choose 3 or fewer items** from a total of 7 choices, how many possible combinations can you select?

$$ {}_7C_3 + {}_7C_2 + {}_7C_1 + {}_7C_0 = 64 $$

3. A hockey team consists of ten offensive players, seven defensive players, and three goaltenders. In how many ways can the coach select a starting line up of three offensive players, two defensive players, and one goaltender?

$$ {}_{10}C_3 \cdot {}_7C_2 \cdot {}_3C_1 = 7560 $$

$$_nC_r = \frac{n!}{(n-r)! \cdot r!} \qquad _nP_r = \frac{n!}{(n-r)!}$$

Mixed Practice: Indicate if the situation following is a Permutation or Combination. Then, solve.

a. In a bingo game 30 people are playing for charity. There are prizes for 1st through 4th. How many ways can we award the prizes?

Permutation or Combination $\quad _{30}P_4 = 657720$

b. From a 30-person club, in how many ways can a President, Treasurer and Secretary be chosen?

Permutation or Combination

$_{30}P_3 = 24360$

$$ {}_nC_r = \frac{n!}{(n-r)! \cdot r!} \qquad {}_nP_r = \frac{n!}{(n-r)!} $$

Mixed Practice: Indicate if the situation following is a Permutation or Combination. Then, solve.

c. In a bingo game 30 people are playing for charity. There are two $50 prizes. In how many ways can prizes be awarded?

Permutation or Combination

$${}_{30}C_2 = 435$$

d. How many 3-digit passwords can be formed with the numbers 1, 2,3,4,5 and 6 if no repetition is allowed?

Permutation or Combination

$${}_6P_3 = 120$$

$$ {}_nC_r = \frac{n!}{(n-r)! \cdot r!} \qquad {}_nP_r = \frac{n!}{(n-r)!} $$

Mixed Practice: Indicate if the situation following is a Permutation or Combination. Then, solve.

e. Converse is offering a limited edition of shoes. They are individually made for you and you choose 4 different colors from a total of 25 colors. How many shoes are possible?

Permutation or Combination

$${}_{25}C_4 = 12650$$

f. A fast food chain is offering a $5 box special. You can choose no more than 5 items from a list of 8 items on a special menu. In how many ways could you fill the box?

Permutation or Combination

$${}_8C_5 + {}_8C_4 + {}_8C_3 + {}_8C_2 + {}_8C_1 + {}_8C_0 = 219$$

Closing

- Ticket out the door
  - Write down the two new formulas you learned.
  - Write down what n! means.