

1. Factor $x^2 + 8x + 15$

Answer: $(x+5)(x+3)$

2. Factor $x^2 - 11x + 24$

Answer: $(x-8)(x-3)$

3. Factor $x^2 + x - 12$

Answer: $(x+4)(x-3)$

4. Factor $3x^2 + 8x + 5$ use "bustin' the b" s

$3x^2 + 3x + 5x + 5$
 $3x(x+1) + 5(x+1)$ Answer: $(3x+5)(x+1)$

Factor And Solve:

5. Solve $(5x-4)(x+3) = 0$

Answer: $x = \frac{4}{5}, -3$

6. Solve $x^2 - 8x + 12 = 0$ $(x-6)(x-2) = 0$

Answer: $6, 2 = x$

7. Solve $x^2 + 12 = 7x$

$x^2 - 7x + 12 = 0$
 $(x-4)(x-3) = 0$

Answer: $x = 4, 3$

8. The quadratic formula is

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

9. A quadratic has

Discriminant

a. 2 real solutions when

$b^2 - 4ac > 0$
 positive

b. 1 real solution when

$b^2 - 4ac = 0$

c. 0 real solutions when

$b^2 - 4ac < 0$
 negative

↳ two imaginary solutions

10. Find the exact value of the solution(s) of

a. $-4x + 3 = x^2$ $x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-3)}}{2(1)}$

$0 = x^2 + 4x - 3$ $x = \frac{-4 \pm \sqrt{28}}{2} = \frac{-4 \pm 2\sqrt{7}}{2} = -2 \pm \sqrt{7}$

b. $3 = 3x^2 + 4x$ $x = \frac{-4 \pm \sqrt{(4)^2 - 4(3)(-3)}}{2(3)} = \frac{-4 \pm \sqrt{52}}{6} = \frac{-4 \pm 2\sqrt{13}}{6}$

11. How many real solutions does each quadratic have?

a. $y = x^2 + x + 5$

$(1)^2 - 4(1)(5) = 1 - 20 = -19$
 0 real, 2 imaginary

b. $y = x^2 + 6x + 9$

$(6)^2 - 4(1)(9) = 0$
 1 real, rational solution

c. $y = x^2 + 6x + 8$

$(6)^2 - 4(1)(8) = 36 - 32 = 4$
 2 real, rational solutions

12. How many times will a parabola touch the x-axis if its quadratic has

a. 2 real solutions two

b. 1 real solution one

c. 0 real solutions zero

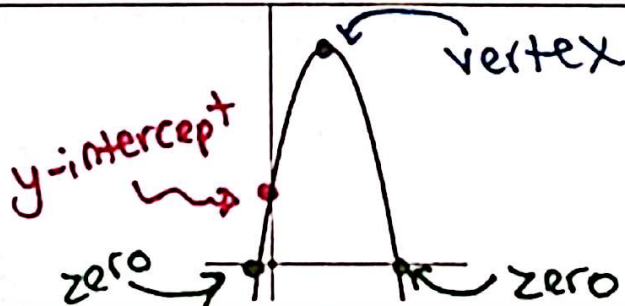
↳ two imaginary solutions

$\frac{-2 \pm \sqrt{13}}{3}$

Graphs of Quadratics

13. Label the graph to show the

y-intercept
 zeros
 vertex



14. To find the x-value of the vertex by hand,

you use the formula $x = \frac{-b}{2a}$.

15. What are two other vocabulary terms for x-intercept?

root, zero.

16. The vertex of $y = -x^2 + 8x - 13$ is at
 $x = \frac{-8}{2(-1)} = \frac{-8}{-2} = 4$ (4, 3)

17. The x-intercepts of $y = x^2 + 2x - 8$ are
 $(x+4)(x-2)$ (-4, 0) and (2, 0)
 $x = -4, 2$

18. A parabola opens up (like a smile) if
 $a > 0$ ("a" is positive)

19. A parabola opens down (like a frown) if
 $a < 0$ ("a" is negative)

20. Which parabolas will open up?
 a. $y = -x^2 + 3x - 5$
b. $y = x^2 - 3x + 5$
c. $y = x^2 + 3x - 5$
 d. $y = -x^2 - 3x + 5$
 } positive "a" values

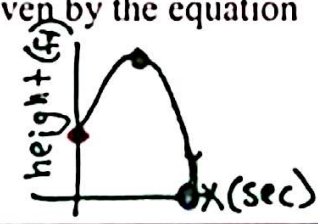
21. Which parabolas will open down?
a. $y = -x^2 + 3x - 5$
 b. $y = x^2 - 3x + 5$
 c. $y = x^2 + 3x - 5$
d. $y = -x^2 - 3x + 5$
 } negative "a" values

22. Determine the amount and type of solutions of $y = -x^2 + 8x - 13$
 Discriminant = $b^2 - 4ac$ determines
 $= (8)^2 - 4(-1)(-13) = 64 - 42 = 22$
 2 real (because \oplus discr.)
 irrational (because non-square discriminant)
 roots

23. Describe how the graph of $y = x^2$ is translated for each equation.
 a. $y = x^2 + 4$ translated up 4
 b. $y = x^2 - 5$ translated down 5
 c. $y = (x - 3)^2$ translated right 3
 d. $y = 3(x + 2)^2$ translated left 2 + stretch
 e. $y = (x + 6)^2 + 2$ translated left 6 and up 2

Applications

24. A rocket is launched into the air. Its height in feet, after x seconds, is given by the equation
 $h(x) = -16x^2 + 300x + 20$.



The starting height of the rocket is 20 ft.
 \hookrightarrow time = $x = 0$ we get y-value of
 The maximum height is 1426 ft.
 \hookrightarrow vertex y-value
 The rocket hits the ground after 18.8 seconds.
 $y = 0$ we find x-intercept

25. Two teenagers throw pennies from the top of the school. The quadratics at the right show how high each penny over time.

Emily: $y = -16x^2 + 20x + 47$
 Isaiah: $y = -16x^2 + 15x + 47$

What are the maximum heights of each penny?
 Emily: 53.25 ft
 Isaiah: 50.52 ft
 When did each penny hit the ground?
 Emily: 2.45 sec
 Isaiah: 2.25 sec

\rightarrow hit the ground when $y = 0$
 so find x-intercept

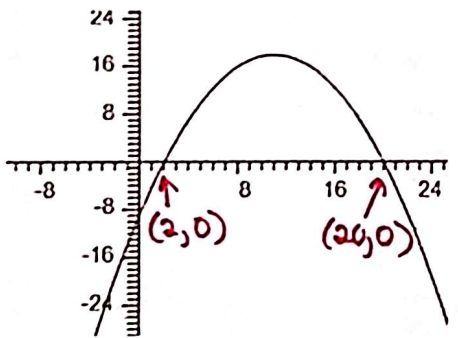
26. Solve the equation by completing the square: $x^2 + 6x = -22$
 $x^2 + 6x + 9 = -22 + 9$
 $\uparrow (b/2)^2$
 $\sqrt{(x+3)^2} = \sqrt{-13}$

27. Find the vertex form of $y = x^2 + 4x + 1$
 $y = x^2 + 4x + 4 - 4 + 1$
 Answer $y = (x+2)^2 - 3$

28. Find the vertex form of $y = -2x^2 + 6x + 1$
 $y = -2(x^2 - 3x + \frac{9}{4} - \frac{9}{4} - \frac{1}{2})$
 $y = -2((x - \frac{3}{2})^2 - \frac{11}{4})$
 $y = -2(x - \frac{3}{2})^2 - \frac{11}{4} \cdot 2$
 Answer $y = -2(x - \frac{3}{2})^2 + \frac{11}{2}$

29. Solve by hand $-4x^2 + 80 = 0$
 $-4x^2 = -80$
 $x^2 = 20$
 $\sqrt{x^2} = \pm\sqrt{20}$
 Answer $\pm 2\sqrt{5}$

30. Write the equation, in standard form, of the parabola in the graph below. The vertex is at (11, 18). Show ALL your work by hand.



$$y = -\frac{2}{9}x^2 + \frac{44}{9}x - \frac{80}{9}$$

$$y = a(x - \text{root})(x - \text{root})$$

$$y = a(x - 2)(x - 20)$$

$$18 = a(11 - 2)(11 - 20)$$

$$18 = a(9)(-9)$$

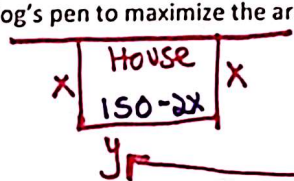
$$\frac{18}{-81} = a \quad a = -\frac{2}{9}$$

$$y = -\frac{2}{9}(x - 2)(x - 20)$$

$$y = -\frac{2}{9}(x^2 - 22x + 40)$$

$$y = -\frac{2}{9}x^2 + \frac{44}{9}x - \frac{80}{9}$$

31. Meg is building a garden up against one side of her house. She has 150 feet of fencing. Find the dimensions of the dog's pen to maximize the area.



perimeter = 150
 $x + y + x = 150$
 $2x + y = 150$
 $y = 150 - 2x$

Area = L · W
 Area = $x(150 - 2x)$
 Area = $150x - 2x^2$
 maximum = vertex
 $(37.5, 2812.5)$
 side = x, y = area

one side = 37.5 ft
 other side = $150 - 2(37.5)$
 side = 75
37.5 by 75 ft

Solve each quadratic inequality. Express your solutions using set notation.

32. $x^2 + 5x \geq 24$
 $x^2 + 5x - 24 \geq 0$
 $(x + 8)(x - 3) = 0$
 $x = -8, 3$
 OR
 Parabola above 0
 $\{x \mid x \leq -8 \text{ or } x \geq 3\}$

In each area, test points
 ① $(-10)^2 + 5(-10) - 24 \geq 0$ ✓
 ② $(0)^2 + 5(0) - 24 \geq 0$ ✗
 ③ $(10)^2 + 5(10) - 24 \geq 0$ ✓
 -8 3

33. $5x^2 + 10 \geq 27x$
 $5x^2 - 27x + 10 \geq 0$
 $5x^2 - 25x - 2x + 10 = 0$
 $5x(x - 5) - 2(x - 5) = 0$
 $(5x - 2)(x - 5) = 0$
 $x = \frac{2}{5}, 5$
 Parabola above 0
 In each area, test points
 ① $5(0)^2 - 27(0) + 10 \geq 0$ ✓
 ② $5(2)^2 - 27(2) + 10 \geq 0$ ✗
 ③ $5(10)^2 - 27(10) + 10 \geq 0$ ✓
 $\{x \mid x \leq \frac{2}{5} \text{ or } x \geq 5\}$

For each of the following, determine the equation for the transformation shown below, from the parent graph $y = x^2$. Then write the equation in standard form.

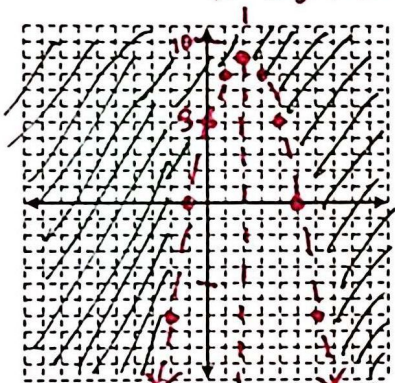
34. Translated left 4, down 3 } $V(-4, -3)$
 vertex form $\rightarrow y = (x+4)^2 - 3$
 $y = (x+4)(x+4) - 3$
 $y = x^2 + 8x + 16 - 3$
 standard form $\rightarrow y = x^2 + 8x + 13$

35. Translated right 3 and } $V(3, 0)$
 reflected over the x-axis $\rightarrow y$'s switch OR sign $a = -1$ (not 1)
 vertex form $\rightarrow y = -(x-3)^2$
 $y = -(x-3)(x-3)$
 $y = -(x^2 - 6x + 9)$
 standard form $\rightarrow y = -x^2 + 6x - 9$

36. Translated left 5, up 2, and } $V(-5, 2)$
 vertically stretched by 3 $\rightarrow a = 3$ (not 1)
 vertex form $\rightarrow y = 3(x+5)^2 + 2$
 $y = 3(x+5)(x+5) + 2$
 $y = 3(x^2 + 10x + 25) + 2$
 $y = 3x^2 + 30x + 75 + 2$
 standard form $\rightarrow y = 3x^2 + 30x + 77$

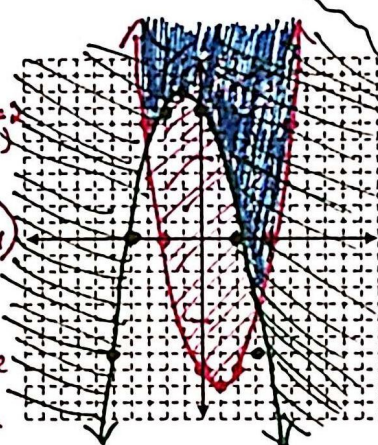
Graph each quadratic inequality or system. Fill in the values requested. Remember to show your work algebraically to receive full credit!

37. $y > -x^2 + 4x + 5$
 $-1(x^2 - 4x - 5)$
 $-1(x-5)(x+1)$
 $x = 5, -1$



x-intercepts: $(-1, 0)$
 vertex: $(2, 9)$ $x = \frac{-4}{2(-1)}$
 is vertex a max or min? \rightarrow max ($a = -1$ so frown \downarrow)
 y-intercept: $(0, 5)$
 AoS: $x = 2$

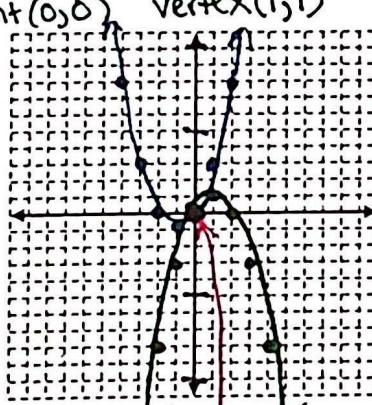
38. $y \geq x^2 - 2x - 8$
 $y \geq -x^2 - 2x + 8$



$(x-4)(x+2)$ x -int $(4, 0)$ $(-2, 0)$
 y -int $(0, 8)$
 vertex $(-1, 9)$
 AoS $x = 1$
 test $(0, 0)$
 $0 \geq 0^2 - 2(0) - 8$ \checkmark
 TRUE
 $0 = -1(x^2 + 2x - 8)$
 $0 = -1(x+4)(x-2)$
 x -int $(-4, 0)$ $(2, 0)$
 y -int $(0, 8)$
 vertex $(-1, 9)$
 AoS $x = -1$
 test $(0, 0)$
 $0 \geq -0^2 - 2(0) + 8$
 $0 \geq 8$ \otimes FALSE

Solve each system of equations. Remember to show your work by hand algebraically to receive full credit!

39. $y = -x^2 + 2x$
 $y = x^2 + 2x$
 $0 = -x(x-2)$ AoS $x = 1$
 x -int $(0, 0)$ $(2, 0)$
 y -int $(0, 0)$ vertex $(1, 1)$



Solution is intersection point $(0, 0)$

40. $y = x^2$
 $y = -x + 2$
 $x^2 + x - 2 = 0$
 $(x+2)(x-1) = 0$
 $x = -2, 1$



Solutions $(-2, 4)$ $(1, 1)$ are intersection points

- 11a. 0, 11b. 1, 11c. 2
 12a. 2, 12b. 1, 12c. 0
 14. use $x = -b/2a$
 15. zero, root
 16. (4, 3)
 17. (-4, 0), (2, 0)
 18. if x^2 is positive
 19. if x^2 is negative
 20. b and c
 21. a and d
 22. 2 real irrational roots
- 23a. up 4
 23b. down 5
 23c. right 3
 23d. 3 times narrower,
 and left 2
 23e. left 6 and up 2

24. starting height = 20 feet
 max height = 1426 feet
 hits ground in 18.8 sec

25. Emily height 53.25 feet
 Isaiah height 50.52 feet
 Emily time 2.45 sec
 Isaiah time 2.25 sec

26. ~~$-3 \pm i\sqrt{3}$~~
 $-3 \pm i\sqrt{13}$

27. $y = (x + 2)^2 - 3$

28. $y = -2(x - 3/2)^2 + 11/2$

29. $2\sqrt{5}$, $-2\sqrt{5}$

30.

$$y = -18/81x^2 + 44/9x - 80/9$$

31. 37.5 ft by 75 ft

32. a. $\{x \mid x \leq -8 \text{ or } x \geq 3\}$

b. $\{x \mid x \leq \frac{2}{5} \text{ or } x \geq 5\}$

Review and Practice of application problems.

1. Which one of these is the standard form of $y = (x-2)^2 + 3$?

a) $y = x^2 + 4x + 7$

b) $y = x^2 - 4x + 7$

c) $y = x^2 - 4x + 4$

d) $y = x^2 + 7$

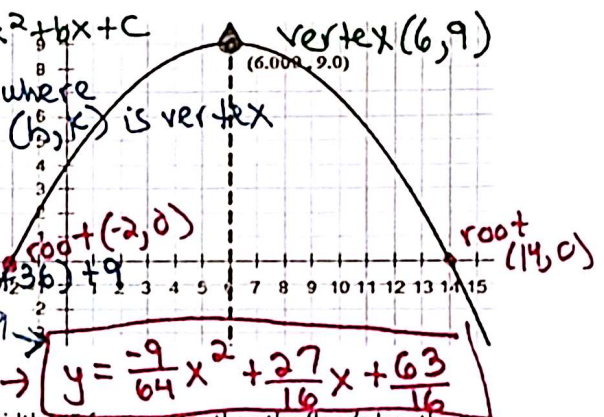
$y = (x-2)(x-2) + 3 = x^2 - 4x + 4 + 3$

2. Write Equation of the Parabola in Standard Form. $y = ax^2 + bx + c$

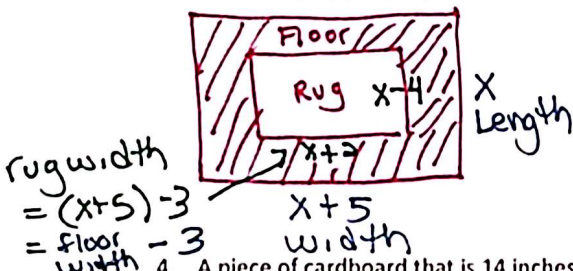
Show ALL work by hand!!

$y = a(x - \text{root})(x - \text{root})$
 $y = a(x+2)(x-14)$
 $9 = a(6+2)(6-14)$
 $9 = a(8)(-8)$
 $\frac{9}{-64} = a$
 $y = \frac{-9}{64}(x+2)(x-14)$

OR $y = a(x-h)^2 + k$ where (h,k) is vertex
 $9 = a(6-6)^2 + 9$
 $0 = a(14-6)^2 + 9$
 $0 = a(8)^2 + 9$
 $0 = \frac{-9}{64}x^2 + \frac{27}{16}x - \frac{28}{16} + 9$
 $y = \frac{-9}{64}x^2 + \frac{27}{16}x + \frac{63}{16}$

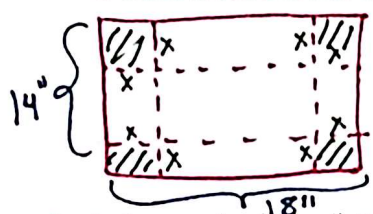


3. A rectangular floor has a rectangular rug on it. The floor's width is 5 feet greater than the floor's length, x . The rug's width is 3 feet less than the floor's width. The rug's length is 6 feet less than the rug's width. Write a function, $R(x)$, in simplified form to represent the area of the floor not covered by the rug.



$R(x) = \text{Area of floor not covered with rug}$
 $R(x) = \text{Area of full floor} - \text{Area of Rug}$
 $R(x) = (x+5)(x) - (x+2)(x-4)$
 $R(x) = x^2 + 5x - (x^2 - 2x - 8)$
 $R(x) = x^2 + 5x - x^2 + 2x + 8$
 $R(x) = 7x + 8$

4. A piece of cardboard that is 14 inches by 18 inches is used to form a box with an open top by cutting away congruent squares with side lengths, x , from the corners. Write an equation y , in terms of x , in standard form to model the surface area of the open box after the corners are cut away.



Surface Area = Area of all surfaces of the figure = Full Rectangle - Area of 4 Congruent Squares
 $\text{Surface Area} = (18)(14) - 4x^2 = 252 - 4x^2$
 $y = \text{Surface Area} = -4x^2 + 252$ (standard form)

5. Each year, a local school's Rock the Vote committee organizes a public rally. Based on previous years, the organizers decided that the Income from ticket sales, $I(t)$ is related to ticket price t by the equation $I(t) = 400t - 40t^2$.

- a. What ticket price(s) would generate the greatest income? What is the greatest income possible? Explain how you obtained the value you got.
 Ticket price(s) \$5.00 Income \$1000.00 $y = I(t) = 400(5) - 40(5)^2 = 2000 - 1000$
 maximum \rightarrow Get vertex $x = \frac{-b}{2a} = \frac{-400}{2(-40)}$
 $x = 5 = t$
- b. At what ticket price(s) would there be no income from the ticket sales. Explain how you obtained the answer.
 $y = 0 \Rightarrow$ find x-intercept

Ticket prices of \$0 and \$10 yield no income

$0 = 400t - 40t^2$
 $0 = -40t^2 + 400t$
 $0 = -40t(t - 10)$
 $t = 0, 10$

There is NO income, or $y = I(t) = 0$, at the x-intercepts. I found those by factoring and setting each factor = 0 to solve.