Unit 6 Day 4

Unit 6 Day 4
6.4 Probability of Mutually Inclusive & Exclusive Events

Complement Probability:
The probability that an event will not occur is

$$P(\text{not } E) = 1 - P(E)$$

Ex: Find the probability, given a deck of cards (total of 9 cards) marked A-I, of picking a card that is NOT a vowel. (A, E, I)

$$P(\text{not vowel}) = 1 - P(\text{vowel}) = 1 - \frac{3}{9} = \frac{6}{9} = \boxed{\frac{2}{3}}$$

Ex: In a 52 card deck, what is the probability of drawing a card other than a black ace?

$$P(\text{not black Ace}) = 1 - P(\text{black ace}) = 1 - \frac{2}{52} = \frac{50}{52} = \boxed{\frac{25}{26}}$$

Practice:
Ex 1-5 on Day 4 worksheet

THEN do "OR" Probability Discovery on Day 4 worksheet

12
12
24
36
48
60
72
84
96
108
120
132
144

Day 4: Probability of Mutually Inclusive and Exclusive Events

Warm-Up:

1. Your I-tunes card has enough for 3 of the 7 songs you want. In how many ways could you pick the songs?

$_7C_3 = \boxed{35}$

2. We use 10 digits in our number system. How many 4-digit "numbers" can be formed if no digits are repeated? (Zero is allowed in any position)

↳ permutation is ok

$_{10}P_4 = \boxed{5040}$ OR $10 \cdot 9 \cdot 8 \cdot 7$

3. Confirm your answer to #2 using the Fundamental Counting Principle.

$10 \cdot 9 \cdot 8 \cdot 7 = \boxed{5040}$

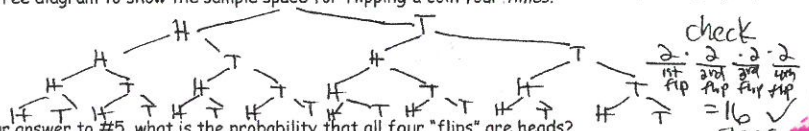
4. Bad Frog Yogurt lets you pick 4 or fewer toppings from 40 choices and save 50 cents off of your order. How many ways can you get the savings?

$_{40}C_4 + {}_{40}C_3 + {}_{40}C_2 + {}_{40}C_1 + {}_{40}C_0$

$91390 + 9880 + 780 + 40 + 1$

$\boxed{102091}$

5. Create a tree diagram to ~~show~~ the sample space for flipping a coin four *times.*



check

$\frac{2}{1\text{st flip}} \cdot \frac{2}{2\text{nd flip}} \cdot \frac{2}{3\text{rd flip}} \cdot \frac{2}{4\text{th flip}} = 16$ items ✓

6. Using your answer to #5, what is the probability that all four "flips" are heads?

$\boxed{1/16}$

Day 4: Probability of Mutually Inclusive and Exclusive Events

Probability of an event NOT occurring

The probability that an event E will not occur is equal to one MINUS **the probability that it will occur.**

$P(E^c) =$ P(not E) = $1 - P(E)$

Ex 1: Find the probability that you choose a number from 1 to ten that is not 6.

$P(\text{not } 6) = 1 - P(6)$

$= 1 - 1/10$ $\boxed{\frac{9}{10}}$

Ex 2: Find the probability that you deal a card that is not a diamond.

$P(\text{not diamond}) = 1 - P(\text{diamond})$

$= 1 - \frac{13}{52} = 1 - \frac{1}{4} = \boxed{3/4}$

Ex 3: You draw a card that is not a red face card (Jack, Queen, King)

$P(\text{not red face}) = 1 - P(\text{red face})$

$= 1 - \frac{6}{52} = \frac{46}{52} = \boxed{\frac{23}{26}}$

Ex 4: You select someone in the class who is not wearing jeans.

$P(\text{not jeans}) = 1 - P(\text{jeans})$

Ex 5 In the classic lottery game, each player chooses 6 different numbers from 1 to 48. If all of the numbers match the 6 picked, they win. What is the probability of not winning?

$_{48}C_6 = 12,271,512$ ways to pick numbers

$P(\text{win}) = \frac{1}{12,271,512}$

$P(\text{not win}) = 1 - P(\text{win})$

$= \frac{12,271,511}{12,271,512}$

$= \boxed{.99999992}$

"OR" Probability Discovery

Part A: Fifteen plastic squares are placed in a box: four blue, seven red, and four brown. If one plastic square is chosen at random from the box, calculate the following:

Blue	Blue	Red	Red	Brown
Blue	Red	Red	Red	Brown
Blue	Red	Red	Brown	Brown

1) P(Blue) $\frac{4}{15}$ 2) P(Red) $\frac{7}{15}$ 3) P(Blue or Red) $\frac{11}{15}$

4) P(Blue) $\frac{4}{15}$ 5) P(Brown) $\frac{4}{15}$ 6) P(Blue or Brown) $\frac{8}{15}$

7) P(Brown) $\frac{4}{15}$ 8) P(Red) $\frac{7}{15}$ 9) P(Brown or Red) $\frac{11}{15}$

10) How could you use the answers in the first two columns to get the answer to the third column? (Example: how could you use #1 and #2 to get #3?)

add 1st 2 columns = 3rd column

11) Suppose you need to write a formula for P(A or B) using P(A), P(B), mathematical symbols, and the information from the problems above. What could the formula be?

$P(A \text{ or } B) = P(A) + P(B)$

Definitions:

Mutually Exclusive: Two or more events that cannot occur at the same time.
Example: a number on a die being even and odd

Mutually Inclusive: Two events that can occur at the same time.
Example: a number on a die being even and less than five (#s in common)

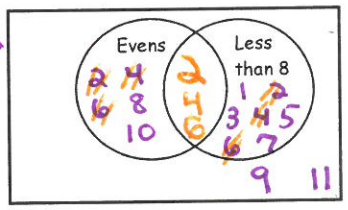
The probability of mutually exclusive scenarios require more thought and careful attention.

Part B: Suppose eleven plastic squares are placed in a box with the numbers 1 – 11 on them. Use this information to calculate the following:

1	2	3	4	5	6
7	8	9	10	11	

1) P(Even) $\frac{5}{11}$ 2) P(Less than 8) $\frac{7}{11}$

3) Create a Venn Diagram for Evens and Less than 8



4) P(Even and Less than 8) $\frac{3}{11}$

5) Even U Less than 8 (remember U means "or" so this means numbers that are even, less than 8, or both) 9

6) P(Even U Less than 8) $\frac{9}{11}$

7) Why does it not make sense to simply add P(Even) and P(Less than 8) to get P(Even U Less than 8) ?

there are shared (overlapping) items

8) How could you have used your answers from P(Even), P(Less than 8) and P(Even and Less than 8) to get P(Even U Less than 8)?

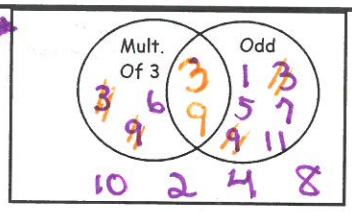
$P(\text{Even} \cup <8) = P(\text{Even}) + P(<8) - P(\text{Even} \& <8)$

$= \frac{5}{11} + \frac{7}{11} - \frac{3}{11} = \frac{9}{11}$ ✓ same as question #6

9) P(Multiple of 3) $\frac{3}{11}$ 10) P(Odd) $\frac{6}{11}$

11) Create a Venn Diagram for Multiple of 3 and Odd

1	2	3	4	5	6
7	8	9	10	11	



12) P(Multiple of 3 ∩ Odd) (Hint: remember ∩ means "and") 3, 9 $\frac{2}{11}$

13) Multiple of 3 U Odd $\frac{7}{11}$

14) P(Multiple of 3 U Odd) $\frac{7}{11}$

15) How could you have used your answers from P(Multiple of 3), P(Odd) and P(Multiple of 3 ∩ Odd) to get P(Multiple of 3 U Odd)?

$P(\text{mult of 3} \cup \text{odd}) = P(\text{mult of 3}) + P(\text{odd}) - P(\text{mult of 3 and odd})$

$3/11 + 6/11 - 2/11 = \frac{7}{11}$ ✓

16) Suppose you need to write a NEW formula for P(A or B), now that you see that your prior formula from Part A will not always work! Using P(A), P(B), P(A ∩ B), mathematical symbols, and the information in the problems, what could be the formula for P(A U B)?

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

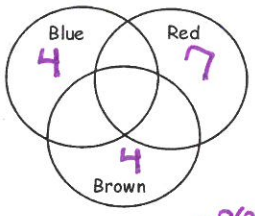
Same as Q#14 ☺

Part C:

Let's take a look back at the scenario from Part A to see if our new formula and methods will work for that scenario too.

Blue	Blue	Red	Red	Brown
Blue	Red	Red	Red	Brown
Blue	Red	Red	Brown	Brown

1) Create a Venn Diagram for blue and red and brown.



2) P(Blue ∩ Red) 0

3) P(Blue ∩ Brown) 0

4) P(Brown ∩ Red) 0

no overlaps!

Use the formula you found at the end of Part B and the work from the past few questions to calculate the following:

5) P(Blue U Red) $= P(B) + P(R) - P(B \cap R)$ $\frac{4}{15} + \frac{7}{15} - 0 = \frac{11}{15}$

6) P(Blue U Brown) $= P(Bl) + P(Br) - P(Bl \cap Br)$ $= \frac{4}{15} + \frac{4}{15} - 0 = \frac{8}{15}$

7) P(Brown U Red) $= P(Br) + P(R) - 0$ $= \frac{4}{15} + \frac{7}{15} - 0 = \frac{11}{15}$

8) Compare the last there answers to your answers for the "OR" probabilities in Part A. They should be the same. If they are not, check your work.

Yes!! ☺ They match!

12
12
24
36
48
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144

"OR" Probability

Mutually Exclusive Events:
events that CANNOT occur at the same time

Ex: Rolling an odd and 2 on 1 roll of the die.

Ex: Drawing 1 card from the deck and it is black and a ♥ heart.

For mutually exclusive events,
$P(A \text{ and } B) = 0$

* Probability of mutually Exclusive Events

$$P(A \text{ or } B) = P(A) + P(B)$$
or
$$P(A \cup B) = P(A) + P(B)$$

Given a spinner marked 1-8, find probability of

a) spinning a 4 or 6 on 1 spin

$P(4 \text{ or } 6) = P(4) + P(6) =$
$1/8 + 1/8 = 2/8 = \boxed{1/4}$

b) spinning an even or a number less than 2 on 1 spin

$P(\text{Even or } <2) = P(\text{even}) + P(<2)$
$\frac{4}{8} + \frac{1}{8} = \boxed{5/8}$

12
12
24
36
48
60
72
84
96
108
120
132
144

Mutually Inclusive Events

events that CAN happen at the same time

Ex: Rolling an odd number and a number less than 4 on 1 roll of a die

* Probability of Mutually Inclusive Events

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

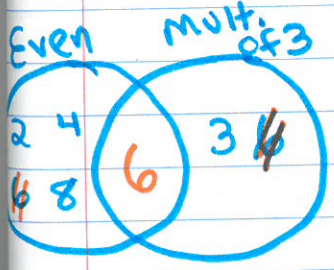
or

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

("overlap")

Ex: Given one spin on a spinner marked 1-8, find the probability of

a) spinning an even number or a multiple of 3

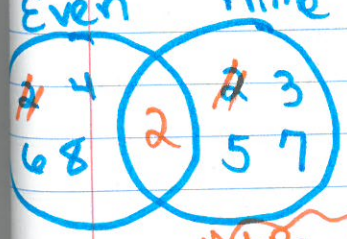


$P(\text{even or mult. of 3}) = P(\text{even}) + P(\text{mult. of 3}) - P(\text{even and mult of 3})$

$P(\text{even}) = \frac{4}{8}$ (2, 4, 6, 8); $P(\text{mult. of 3}) = \frac{2}{8}$ (3, 6); $P(\text{even and mult of 3}) = \frac{1}{8}$ (6)

$$\frac{4}{8} + \frac{2}{8} - \frac{1}{8} = \boxed{\frac{5}{8}}$$

b) spinning an even number or a prime number



$P(\text{even or prime}) = P(\text{even}) + P(\text{prime}) - P(\text{even and prime})$

$P(\text{even}) = \frac{4}{8}$ (2, 4, 6, 8); $P(\text{prime}) = \frac{4}{8}$ (2, 3, 5, 7); $P(\text{even and prime}) = \frac{1}{8}$ (2)

$$= \frac{4}{8} + \frac{4}{8} - \frac{1}{8} = \boxed{\frac{7}{8}}$$

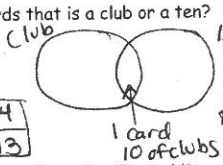
** Practice on Right side of WKST **

Examples:

1. What is the probability of choosing a card from a deck of cards that is a club or a ten?

P(choosing a club or a ten) = $P(\text{club}) + P(10) - P(\text{club} + 10)$ (overlap)

$\frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \boxed{\frac{4}{13}}$

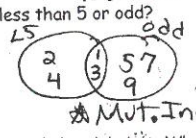


* Mut. Inclusive

2. What is the probability of choosing a number from 1 to 10 that is less than 5 or odd?

$P(<5) + P(\text{odd}) - P(<5 \text{ and odd})$

$\frac{4}{10} + \frac{5}{10} - \frac{2}{10} = \boxed{\frac{7}{10}}$

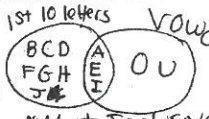


* Mut. Inclusive

3. A bag contains 26 tiles with a letter on each, one tile for each letter of the alphabet. What is the probability of reaching into the bag and randomly choosing a tile with one of the first 10 letters of the alphabet on it or randomly choosing a tile with a vowel on it?

$P(\text{1st 10 letters or vowel}) = P(\text{1st 10 letters}) + P(\text{vowel}) - P(\text{1st 10 and vowel})$

$= \frac{10}{26} + \frac{5}{26} - \frac{3}{26} = \frac{12}{26} = \boxed{\frac{6}{13}}$



* Mut. Inclusive

4. A bag contains 26 tiles with a letter on each, one tile for each letter of the alphabet. What is the probability of reaching into the bag and randomly choosing a tile with one of the last 5 letters of the alphabet on it or randomly choosing a tile with a vowel on it?

$P(\text{last 5 letters or vowel}) - \bigcirc$ (no overlap to subtract away)

$= P(\text{last 5 letters}) + P(\text{vowel}) - \bigcirc$

$= \frac{5}{26} + \frac{5}{26} = \frac{10}{26} = \boxed{\frac{5}{13}}$

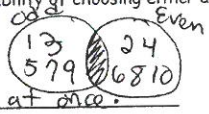
* mut. exclusive

no need to subtract out the overlap

EX. Spinning a 4 or 6 on a 1-8 spinner (numbers 1, 2, 3, 4, 5, 6, 7, and 8 occur on the spinner). Are they mutually exclusive or mutually inclusive? Mutually exclusive $P(4 \text{ or } 6) = \frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \boxed{\frac{1}{4}}$

Examples: Extra Example: Prop brown hair or green eyes in class $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

1. If you randomly chose one of the integers 1 – 10, what is the probability of choosing either an odd number or an even number?



Are these mutually exclusive events? Why or why not?

yes. A number can't be even and odd at once.

Complete the following statement: P(odd or even) = P(odd) + P(even)

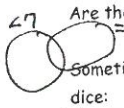
Now fill in with numbers: P(odd or even) = $\frac{5}{10} + \frac{5}{10} = 1$

Does this answer make sense?

yes. A number must be even or odd, so since these are the only 2 possibilities, their sum should be 1 for 100%.

2. Two fair dice are rolled. What is the probability of getting a sum less than 7 or a sum equal to 10?

Are these events mutually exclusive? yes. Sum can't be < 7 and = 10 because 10 is greater than 7



Sometimes using a table of outcomes is useful. Complete the following table using the sums of two dice:

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

P(getting a sum less than 7 OR sum of 10) = $P(\text{sum} < 7) + P(\text{sum} = 10) = \frac{15}{36} + \frac{3}{36}$

This means half of the time we roll two dice, their sum should be < 7 or = 10. $= \frac{18}{36} = \boxed{\frac{1}{2}}$