Unit 4B Day 3 and 4

Notes on CPCTC
and proofs with overlapping triangles

Day 3 Warm-Up

33. Which of the following is NOT a method used to prove triangles congruent?

A. AAS **B.** ASA **C.** SAS **D.** SSA

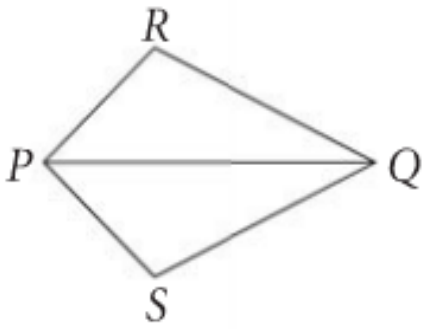
34. Suppose $\overline{RT} \cong \overline{ND}$ and $\angle R \cong \angle N$. What additional information is needed to prove $\triangle RTJ \cong \triangle NDF$ by ASA?

F. $\angle T \cong \angle D$ **G.** $\angle R \cong \angle N$ **H.** $\angle J \cong \angle D$ **J.** $\angle T \cong \angle F$

35. $\overline{PQ}$ bisects $\angle RPS$ and $\angle RQS$. Justify each answer.

a. Which pairs of angles, if any, are congruent?

b. By what theorem or postulate can you prove that $\triangle PRQ \cong \triangle PSQ$?

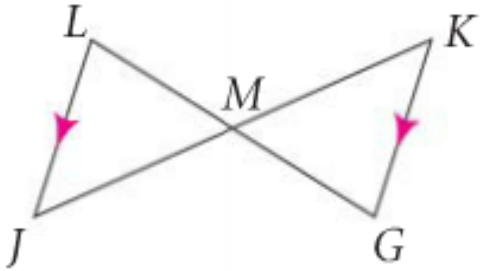


36. $\overline{LJ} \parallel \overline{KG}$ and M is the midpoint of $\overline{LG}$.

a. Why is $\overline{LM} \cong \overline{GM}$?

b. Can the two triangles be proved congruent by ASA? Explain.

c. Can the two triangles be proved congruent by AAS? Explain.



Then do # 26, 28, 30 from yesterday's Proof Packet

Day 3 Warm-Up Answers

ASA

33. Which of the following is NOT a method used to prove triangles congruent?

A. AAS B. ASA C. SAS D. SSA

34. Suppose $\overline{RT} \cong \overline{ND}$ and $\angle R \cong \angle N$. What additional information is needed to prove $\triangle RTJ \cong \triangle NDF$ by ASA?

F. $\angle T \cong \angle D$ G. $\angle R \cong \angle N$ H. $\angle J \cong \angle D$ J. $\angle T \cong \angle F$

35. $\overline{PQ}$ bisects $\angle RPS$ and $\angle RQS$. Justify each answer.

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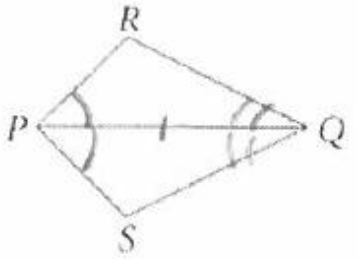


36. $\overline{LJ} \parallel \overline{KG}$ and M is the midpoint of $\overline{LG}$.

a. Why is $\overline{LM} \cong \overline{GM}$?

b. Can the two triangles be proved congruent by ASA? Explain. Yes

c. Can the two triangles be proved congruent by AAS? Explain. Yes

HW Discussion

4-4 Using Congruent Triangles: CPCTC

Proving Parts of Triangles Congruent:

With SSS, SAS, ASA, and AAS, you know how to use three parts of triangles to show that triangles are congruent.

Once you have triangles congruent, you can make conclusions about their other parts.

By definition, corresponding parts of congruent triangles are congruent.

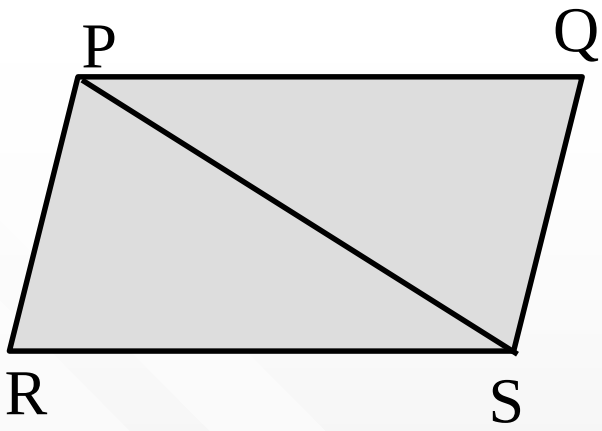
You can abbreviate this as **CPCTC**

Ex 1.

Given: $\angle R \cong \angle Q$

$\angle QPS \cong \angle RSP$

Prove: $\overline{PR} \cong \overline{SQ}$

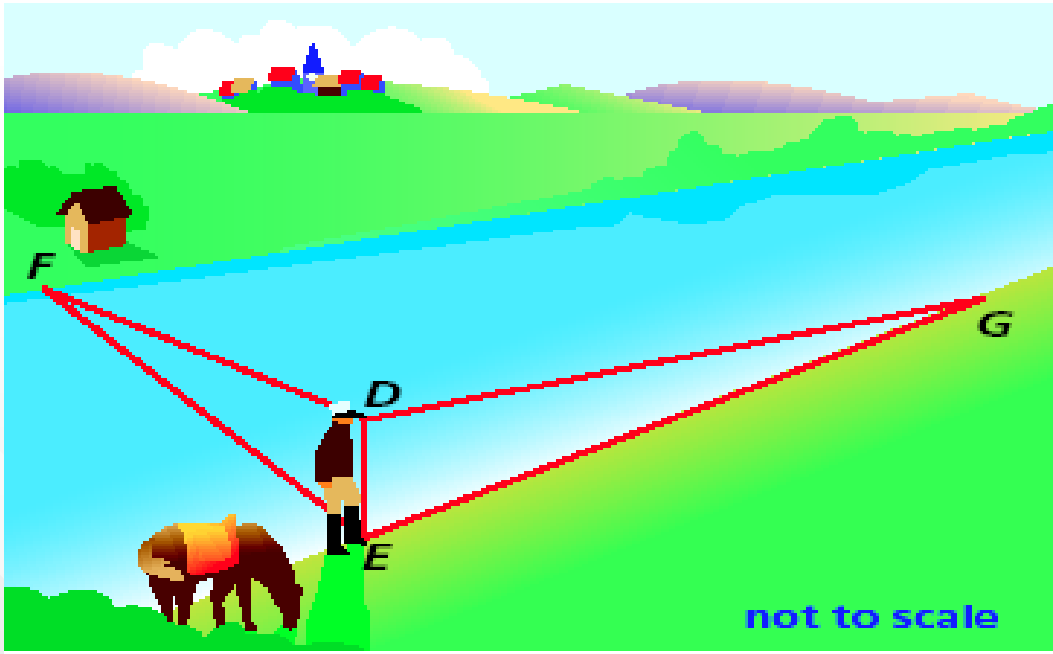


Statement	Reason
1) $\angle R \cong \angle Q$, $\angle QPS \cong \angle RSP$	1) Given
2) $\overline{PS} \cong \overline{PS}$	2) Reflexive property of congruence
3) $\Delta QPS \cong \Delta RSP$	3) AAS Theorem
4) $\overline{PR} \cong \overline{SQ}$	4) CPCTC

What other pairs of sides and angles can you conclude are congruent by CPCTC?

Example

According to legend, one of Napoleon's officer used congruent triangles to estimate the width of a river. On the riverbank, the officer stood up straight and lowered the visor of his cap until the farthest thing he could see was the edge of the opposite bank. He then turned and noted the spot on his side of the river that was in line with his eye and the tip of his visor. The officer then paced off the distance to this spot and declared that distance to be the width of the river! Use congruent triangles to prove that he was correct.



Given:

$\angle DEG$ and $\angle DEF$ are right angles

$\angle EDG \cong \angle EDF$

Prove:

$\overline{EG} \cong \overline{EF}$ (proof on next slide)

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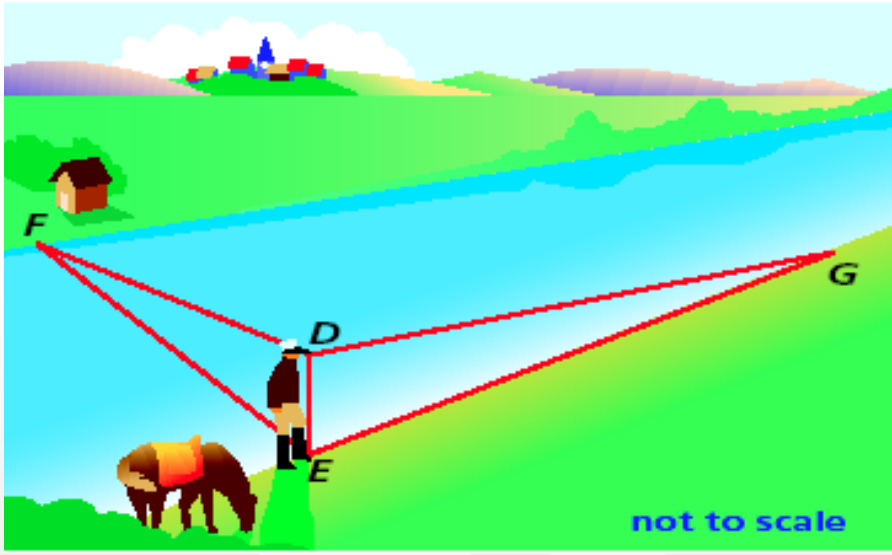
$\angle DEG$ and $\angle DEF$ are right angles

$\angle EDG \cong \angle EDF$

Prove:

$\overline{EG} \cong \overline{EF}$



Statement	Reasons
1. $\angle EDG \cong \angle EDF$	**1.** Given
2. $\overline{DE} \cong \overline{DE}$	**2.** Reflexive Property of Congruence
3. $\angle DEG$ and $\angle DEF$ are right angles.	**3.** Given
4. $\angle DEG \cong \angle DEF$	**4.** All right angles are congruent.
5. $\Delta DEF \cong \Delta DEG$	**5.** ASA Postulate
6. $\overline{EG} \cong \overline{EF}$	**6.** CPCTC

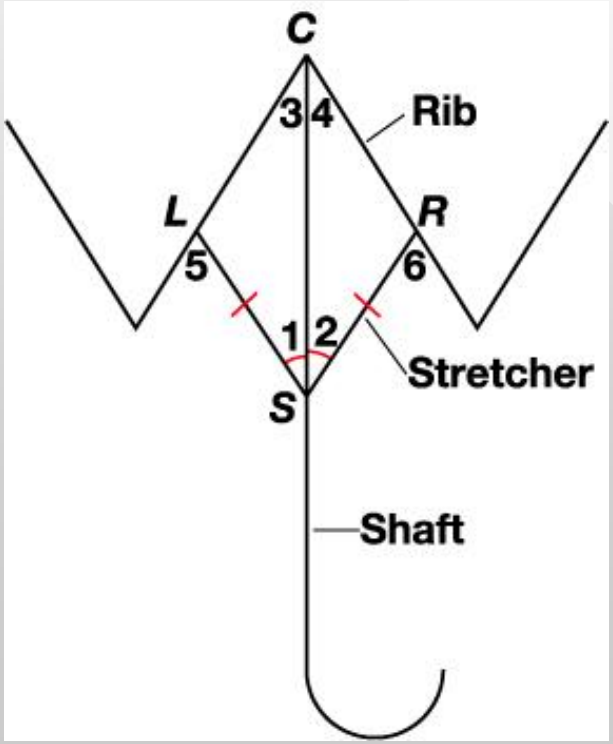
Example

What other congruence statements can you prove from the diagram, in which $\overline{SL} \cong \overline{SR}$, and $\angle 1 \cong \angle 2$ are given?

$\overline{SC} \cong \overline{SC}$ by the Reflexive Property of Congruence,

and $\Delta LSC \cong \Delta RSC$ by SAS Postulate.

Then, $\angle 3 \cong \angle 4$ because corresponding parts of congruent triangles are congruent.



When two triangles are congruent, you can form congruence statements about three pairs of corresponding angles and three pairs of corresponding sides.

You could also use CPCTC to prove

$\angle CLS \cong \angle CRS$ *and* $\overline{CL} \cong \overline{CR}$

Practice

1) **Notes online p. 16 #11, 12, 15**
2) **Bottom 4 Congruent Triangle Challengers from**

http://feromax.com/cgi-bin/ProveIt.pl

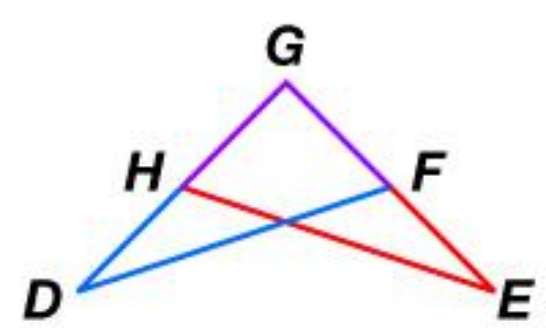
3) **Notes online p. 16-18 #20, 21**

4-7 Using Corresponding Parts of Congruent Triangles

Some triangle relationships are difficult to see because the triangles overlap. Overlapping triangles may have a common side or angle.

You can simplify your work with overlapping triangles by separating and redrawing the triangles.

Ex: Name the parts of their sides that ΔDFG and ΔEHG share.



1st, Identify the overlapping triangles.

2nd, Identify the shared parts.

These are $\overline{HG}$ and $\overline{FG}$, respectively, and angle G.

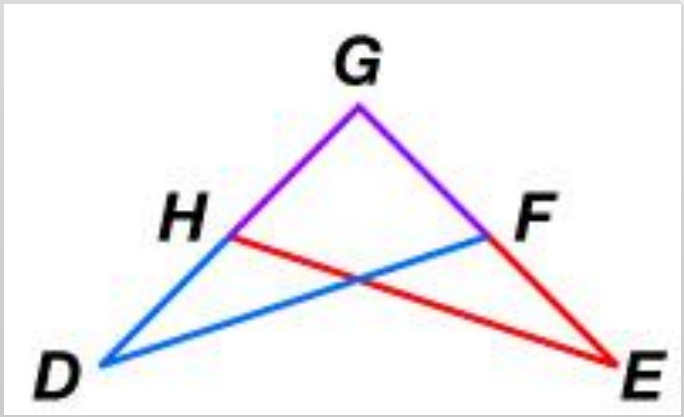
More tips on the next slide! ☺

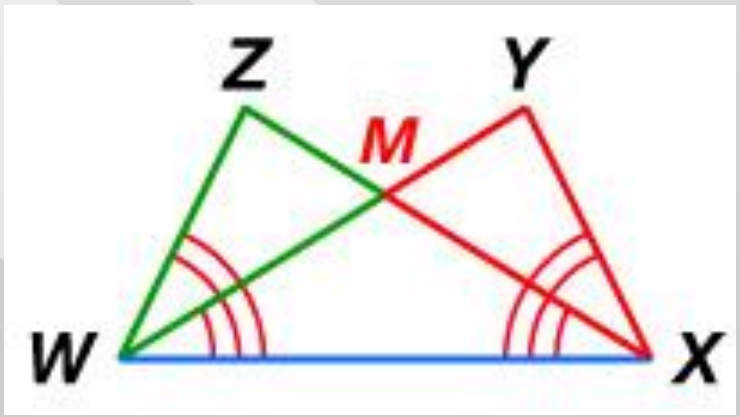
Tips for identifying reflexive parts

Draw the triangles separately. Using colors can help! Trace the triangles on the original diagram. Look at where the colors overlap.

Then, look at the amount of repeated letters between the two triangles.

- **Often, 1 repeated letter means there is an angle pair congruent by reflexive property.**
- **Often, 2 repeated letters means there is a side pair congruent by reflexive property.**



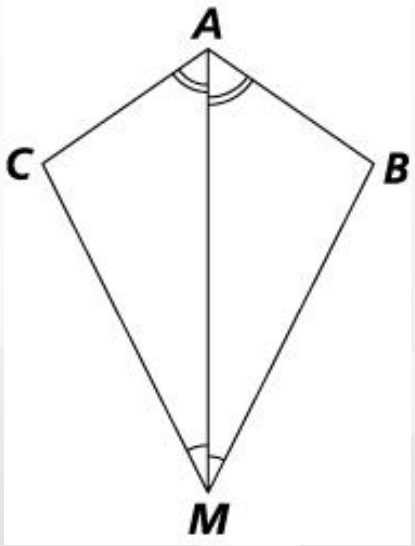


Practice

Continue work on online Notes p. 15-18

Exit Ticket:

1. What does “CPCTC” stand for?



Use the diagram for Exercises 2 and 3.

2. Tell how you would show

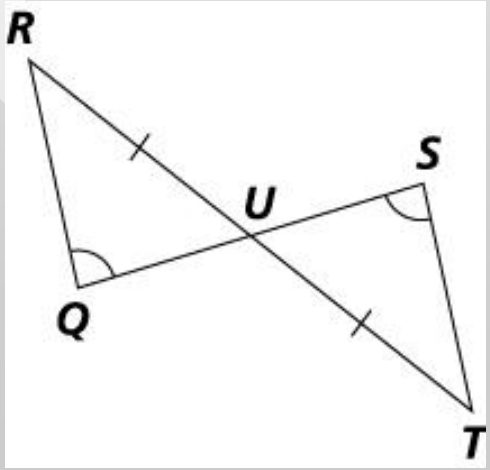
$\triangle ABM \cong \triangle ACM$.

3. Tell what other parts are congruent by CPCTC.

Use the diagram for Exercise 4.

4. Given: $\angle Q \cong \angle S$, and $\overline{RU} \cong \overline{TU}$

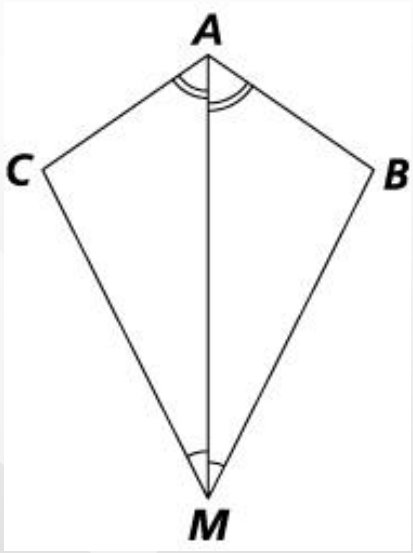
Prove: $\overline{RQ} \cong \overline{TS}$.



1. What does “CPCTC” stand for?

Corresponding parts of congruent triangles are congruent.

Use the diagram for Exercises 2 and 3.



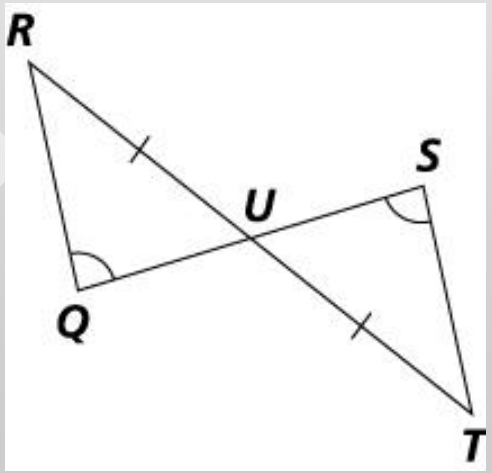
2. Tell how you would show $\triangle ABM \cong \triangle ACM$.

You are given two pairs of $\cong\angle$s and $\overline{AM} \cong \overline{AM}$ by the Reflexive Prop., so $\triangle ABM \cong \triangle ACM$ by ASA.

3. Tell what other parts are congruent by CPCTC.

$\overline{AB} \cong \overline{AC}$, $\overline{BM} \cong \overline{CM}$, $\angle B \cong \angle C$

Use the diagram for Exercises 4 and 5.



4. Given: $\angle Q \cong \angle S$, and $\overline{RU} \cong \overline{TU}$

Prove: $\overline{RQ} \cong \overline{TS}$.

1) $\angle Q \cong \angle S$, and $\overline{RU} \cong \overline{TU}$	1) Given
2) $\angle RUQ \cong \angle TUS$	2) vertical angles are $\cong$
3) $\triangle RUQ \cong \triangle TUS$	3) AAS Theorem
4) $\overline{RQ} \cong \overline{TS}$.	4) CPCTC

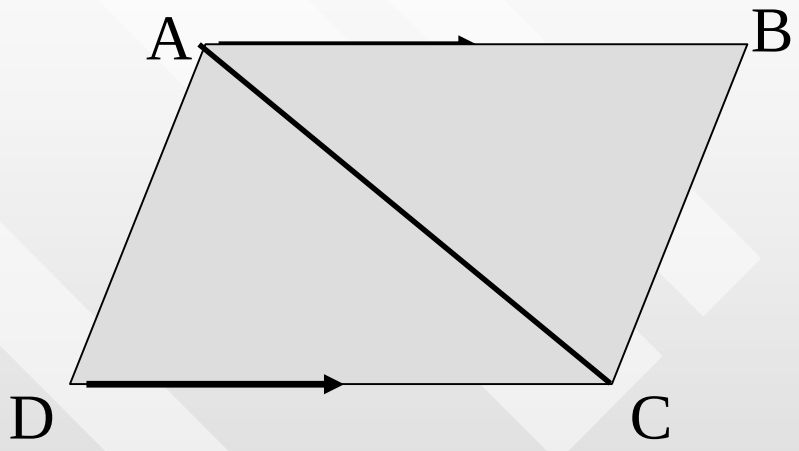
Day 3 Collected Proof:

Requirements: Write down ALL parts of the problem and mark in your picture. No Talking! This proof will be graded <u>for accuracy</u>!

Given: $\overline{AB} \cong \overline{CD}$

$\overline{AB} \parallel \overline{CD}$

Prove: $\triangle ABC \cong \triangle CDA$



Statements	Reasons

Day 3 Collected Proof:

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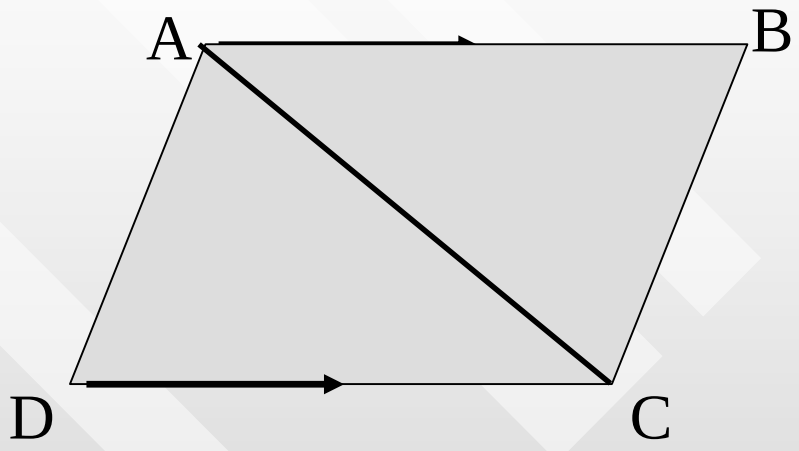


Statements	Reasons

Day 4

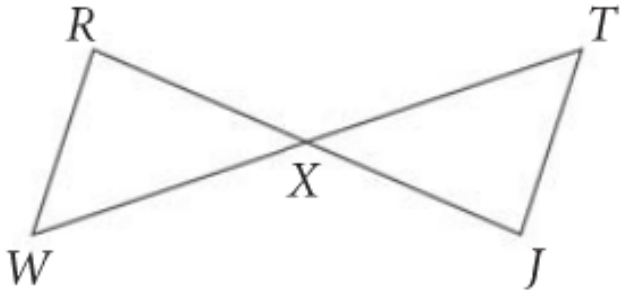
Practice with Proofs

Day 4 Warm-Up

22. In the diagram, $\triangle RXW \cong \triangle JXT$. Which statement is NOT necessarily true?

A. $\angle J \cong \angle R$ **B.** $\angle W \cong \angle T$

C. $\overline{WX} \cong \overline{JX}$ **D.** $\overline{RW} \cong \overline{JT}$



23. Which is true by CPCTC?

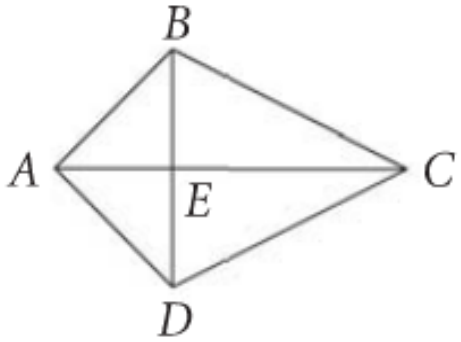
F. $\overline{AC}$ bisects $\overline{BD}$ **G.** $\angle BAC \cong \angle DCA$

H. $\angle ABE \cong \angle EDC$ **J.** $\overline{BC} \cong \overline{DC}$

24. Which is *not* true by CPCTC?

A. $\overline{BE} \cong \overline{DE}$ **B.** $\angle BAC \cong \angle DAC$

C. $\angle BCA \cong \angle DCE$ **D.** $\overline{AB} \cong \overline{AD}$



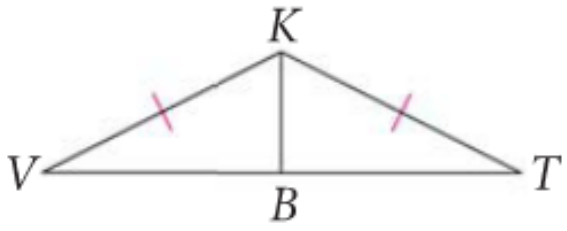
$\triangle ABC \cong \triangle ADC$

Exercises 23–24

25. In the diagram, $\overline{KB}$ bisects $\angle VKT$ and $\overline{KV} \cong \overline{KT}$.

a. What do you need to show in order to conclude $\angle KBV \cong \angle KBT$? State whether it is possible to show this and justify your answer.

b. Prove that $\overline{VB} \cong \overline{TB}$.

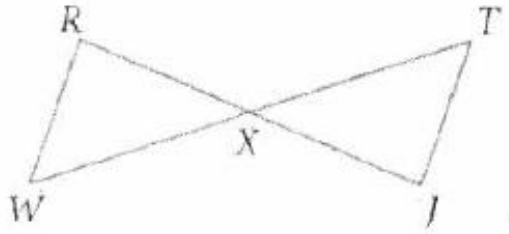


Day 4 Warm-Up Answers

22. In the diagram, $\triangle RXW \cong \triangle JXT$. Which statement is NOT necessarily true?

A. $\angle J \cong \angle R$ B. $\angle W \cong \angle T$

C. $\overline{WX} \cong \overline{JX}$ D. $\overline{RW} \cong \overline{JT}$



23. Which is true by CPCTC?

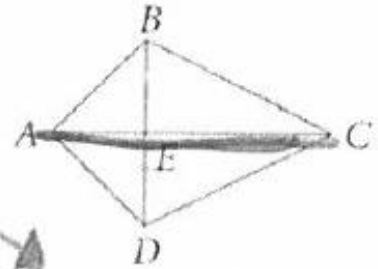
F. $\overline{AC}$ bisects $\overline{BD}$ G. $\angle BAC \cong \angle DCA$

H. $\angle ABE \cong \angle EDC$ J. $\overline{BC} \cong \overline{DC}$

24. Which is *not* true by CPCTC?

A. $\overline{BE} \cong \overline{DE}$ B. $\angle BAC \cong \angle DAC$

C. $\angle BCA \cong \angle DCE$ D. $\overline{AB} \cong \overline{AD}$



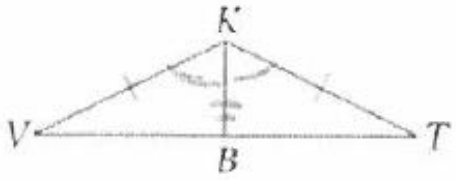
$\triangle ABC \cong \triangle ADC$

Exercises 23–24

25. In the diagram, $\overline{KB}$ bisects $\angle VKT$ and $\overline{KV} \cong \overline{KT}$.

a. What do you need to show in order to conclude $\angle KBV \cong \angle KBT$? State whether it is possible to show this and justify your answer.

b. Show that $\overline{VB} \cong \overline{TB}$.



The $\triangle$s are $\cong$

Yes

Reflexive

SAS

CPCTC

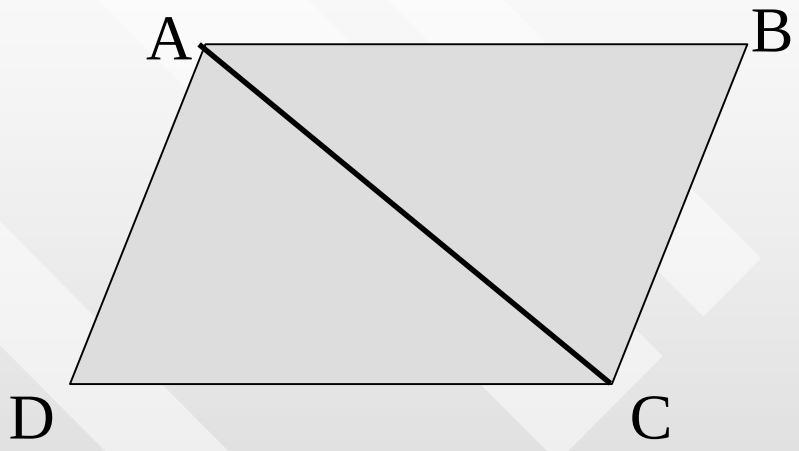
Day 4 Collected Proof Answers:

Requirements: Write down ALL parts of the problem and mark in your picture. No Talking! This proof will be graded <u>for accuracy</u>!

Given: $\overline{AB} \cong \overline{CD}$

$\overline{AB} \parallel \overline{CD}$

Prove: $\overline{AD} \cong \overline{CB}$



Statements	Reasons

Day 4 Collected Proof:

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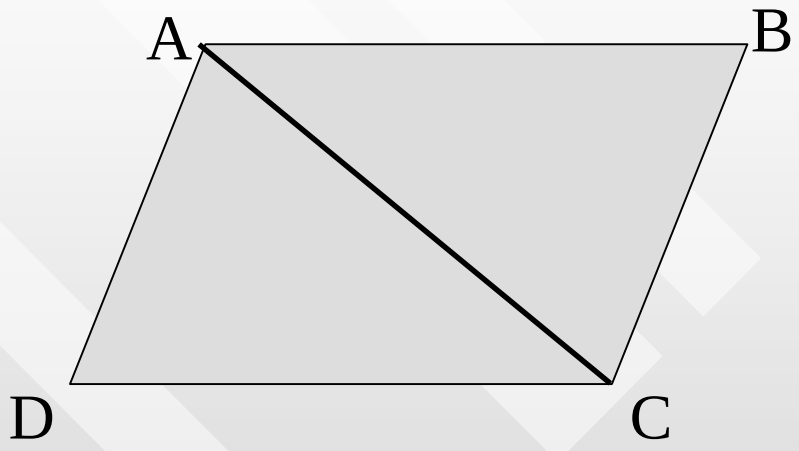
Given: $\overline{AB} \cong \overline{CD}$

$\overline{AB} \parallel \overline{CD}$

Prove: $\overline{AD} \cong \overline{CB}$



Statements	Reasons

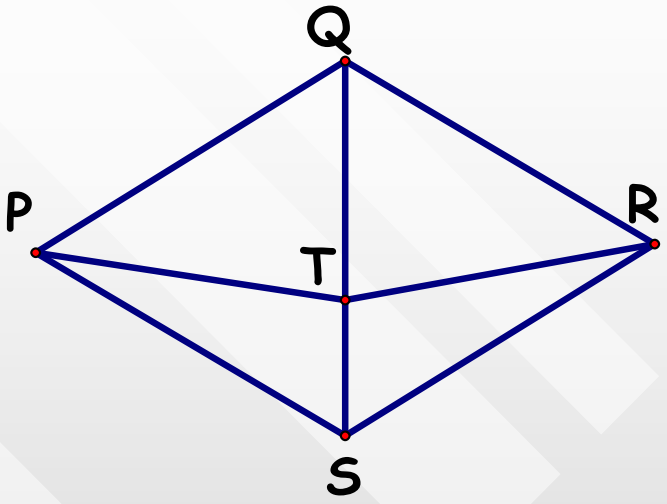
HW Discussion

A tougher CPCTC problem

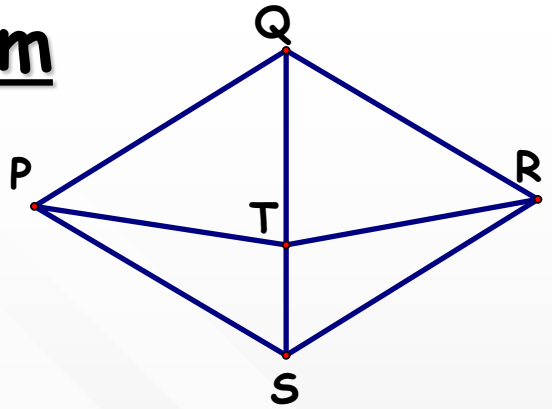
Complete a 2-column proof for….

2) Given: $\overline{PS} \cong \overline{RS}$, $\angle PSQ \cong \angle RSQ$

Prove: $\Delta QPT \cong \Delta QRT$



Answer to tougher CPCTC problem



2) Given: $\overline{PS} \cong \overline{RS}$, $\angle PSQ \cong \angle RSQ$

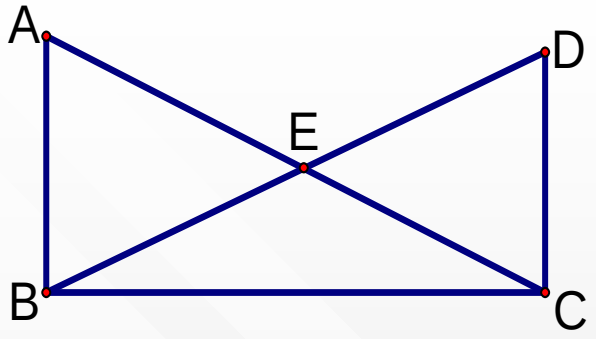
Prove: $\triangle QPT \cong \triangle QRT$

Statement	Reason
$\overline{PS} \cong \overline{RS}$, $\angle PSQ \cong \angle RSQ$	Given
$\overline{SQ} \cong \overline{SQ}$	Reflexive property of congr.
$\Delta PSQ \cong \Delta RSQ$	SAS Post.
$\overline{PQ} \cong \overline{RQ}$ and $\angle PQS \cong \angle RQS$	CPCTC
$\overline{QT} \cong \overline{QT}$	Reflexive property of congr.
$\Delta QPT \cong \Delta QRT$	SAS Post.

Practice Proof – tougher one using CPCTC

Given: $\overline{AB} \perp \overline{BC}$, $\overline{DC} \perp \overline{BC}$, $\overline{AC} \cong \overline{DB}$

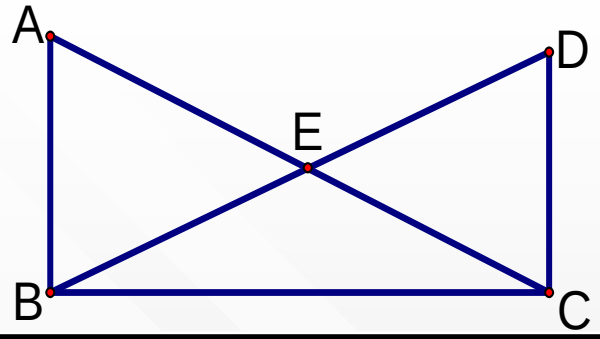
Prove: $\overline{AE} \cong \overline{DE}$



Practice Proofs – tougher one using CPCTC

Given: $\overline{AB} \perp \overline{BC}$, $\overline{DC} \perp \overline{BC}$, $\overline{AC} \cong \overline{DB}$

Prove: $\overline{AE} \cong \overline{DE}$



Statement	Reason
$\overline{AB} \perp \overline{BC}, \overline{DC} \perp \overline{BC}, \overline{AC} \cong \overline{DB}$	Given
$\angle$ABC and $\angle$DCB are Right Angles	Defn. of perpendicular
ΔABC and ΔDCB are Right Δs	Defn. of Right Triangle
$\overline{BC} \cong \overline{BC}$	Reflexive Property of $\cong$
$\Delta ABC \cong \Delta DCB$	HL Theorem
$\overline{AB} \cong \overline{DC}$ and $\angle A \cong \angle D$	CPCTC
$\angle AEB \cong \angle DEC$	Vertical angles Theorem
$\Delta AEB \cong \Delta DEC$	AAS Theorem
$\overline{AE} \cong \overline{DE}$	CPCTC

Practice

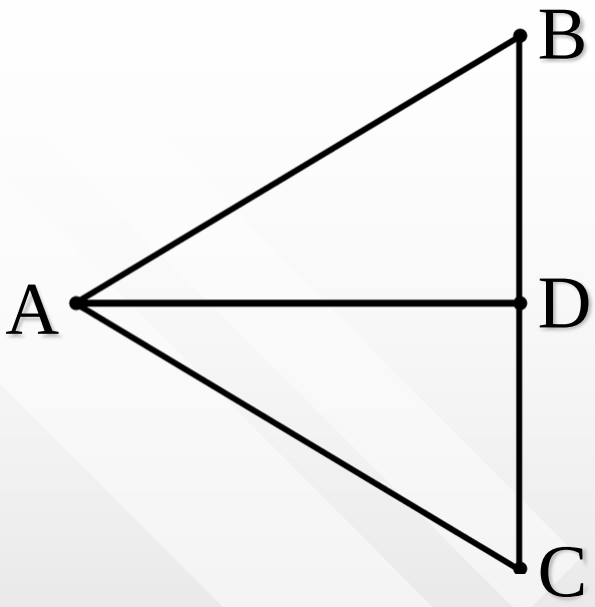
Continue work on online Notes p. 15-18

Day 4 Collected Proof:

Complete a 2-column proof.

Given:

$\overline{AD}$ is a $\perp$ bisector of $\overline{BC}$



Prove:

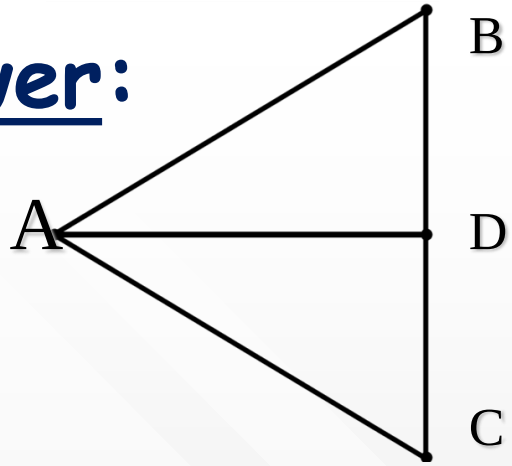
$\overline{AB} \cong \overline{AC}$

Requirements: Write down ALL parts of the problem and mark in your picture. No Talking!

Day 4 Collected Proof Answer:

Given: $\overline{AD}$ is a $\perp$ bisector of $\overline{BC}$

Prove: $\overline{AB} \cong \overline{AC}$



Statement	Reason
$\overline{AD}$ *is a* $\perp$ *bisector of* $\overline{BC}$	Given
$\angle ADC$ and $\angle ADB$ are right $\angle$s	Defn. of perpendicular lines
$\angle ADC \cong \angle ADB$	All Right angles are congruent
$\overline{BD} \cong \overline{CD}$	Defn. of bisector
$\overline{AD} \cong \overline{AD}$	Reflexive Property of congruence
$\angle ADC \cong \angle ADB$	SAS Postulate
$\overline{AB} \cong \overline{AC}$	CPCTC