

## Unit 6 Day 2

### 6.2 Basic Probability, Sets, and Sample Spaces

Sample Space: lists all the possible outcomes of a given experiment

Ex: What are the elements of set A, the factors of 12?

$$A = \{1, 2, 3, 4, 6, 12\}$$

Ex: What is sample space for set B, the factors of 16?

$$B = \{1, 2, 4, 8, 16\}$$

Intersection of two sets:  $A \cap B$

all the elements that appear in BOTH sets A and B

↑  $A \cap B$   
"A and B"

Ex: Find  $A \cap B$  from above

$$\{1, 2, 4\}$$

Union of two sets:  $A \cup B$

all the elements that appear in EITHER A or B (or both)

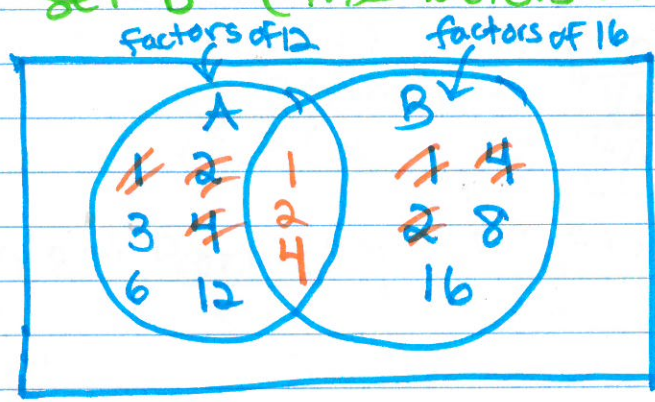
"A OR B"

Ex: Find  $A \cup B$  from above

$$\{1, 2, 3, 4, 6, 8, 12, 16\}$$

Venn Diagram: a diagram that shows all possible relationships between a collection of sets

Ex: Create a Venn Diagram for set A (the factors of 12) and set B (the factors of 16)



Ex: Find  $A \cup B$   
 $\{1, 2, 3, 4, 6, 8, 12, 16\}$

Ex: Find  $A \cap B$   
 $\{1, 2, 4\}$

Complement of a set A:  $A^c$   
 all the elements of the universal set that are NOT in the initial set A

Ex:  $S = \{\dots, \underline{-3}, \underline{-2}, \underline{-1}, \underline{0}, \underline{1}, \underline{2}, \underline{3}, \underline{4}, \dots\}$

and  $A = \{\dots, -2, 0, 2, 4, \dots\}$

If  $A$  is a subset of  $S$ , find  $A^c$ .

$S = \text{integers}$   
 $A = \text{even integers}$

$A^c = \text{NOT } A$   
 $= \text{NOT evens} = \text{odds}$   
 $A^c = \{\dots, -3, -1, 1, 3, 5, \dots\}$

Practice Day 2: Basic Probability

Example 1: Given the following sets, find  $A \cap B$  and  $A \cup B$ .

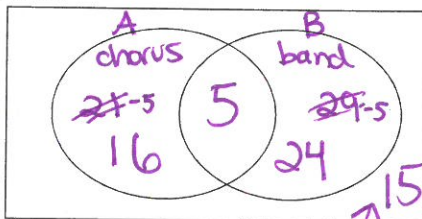
$A = \{1, 3, 5, 7, 9, 11, 13, 15\}$     $B = \{0, 3, 6, 9, 12, 15\}$

$A \cap B = \{3, 9, 15\}$   
both

$A \cup B = \{0, 1, 3, 5, 6, 7, 9, 11, 12, 13, 15\}$   
either

Example 2: In a class of 60 students, 21 sign up for chorus, 29 sign up for band, and 5 take both. 15 students in the class are not enrolled in either band or chorus.

- a. Put this information into a Venn Diagram. If the sample space,  $S$ , is the set of all students in the class, let students in chorus be set  $A$  and students in band be set  $B$ .



- b. What is  $A \cup B$ ? 45  
 c. What is  $A \cap B$ ? 5  
 d. What is  $A^c$ ? 39    $B^c$ ? 31  
 e. What is  $(A \cap B)^c$ ? 55  
 f. What is  $(A \cup B)^c$ ? 15

$60 - 16 - 5 - 24 = 15$

Basic Probability: P(E)  
Probability of Event E

$$P(E) = \frac{\text{\# of ways Event E can occur}}{\text{total \# of outcomes}} = \frac{\text{desired}}{\text{total}}$$

Ex: Given 10 cards labeled 1-10,  
find

a) P(4)	=	$\frac{1}{10}$	
b) P(even)	=	$\frac{1}{2}$	$\frac{5}{10}$
c) P(not 4)	=	$\frac{9}{10}$	

Complement Probability: P(A<sup>c</sup>) or P(Not A)

Remember A<sup>c</sup> is every outcome except A (or NOT A) so

$$P(A^c) = P(\text{not } A) = 1 - P(A)$$

Ex: Find P(not 4) =  
 $1 - P(4) = 1 - \frac{1}{10} = \frac{9}{10}$

Ex: Find P(Factor of 3)<sup>c</sup>  
 $= 1 - P(\text{Factor of } 3) = 1 - \frac{2}{10} = \frac{8}{10}$

Odds: The odds of an event occurring are equal to a ratio

Ex: find odds of

a) ten

$$1:9$$

b) factors of 3

$$2:8 \text{ or } 1:4$$

c) odds

$$5:5 \text{ or } 1:1$$

favorable outcomes : not favorable outcomes  
# of successes : # of failures

Example 3: A spinner has 4 equal sectors colored yellow, blue, green and red. After spinning the spinner, what is the probability of landing on each color?

P(yellow) =  $\frac{1}{4}$

P(green) =  $\frac{1}{4}$

P(blue) =  $\frac{1}{4}$

P(red) =  $\frac{1}{4}$

Example 4: A single 6-sided die is rolled. What is the probability of each outcome?  
What is the probability of rolling an even number? Of rolling an odd number?

P(1) =  $\frac{1}{6}$  P(2) =  $\frac{1}{6}$  P(3) =  $\frac{1}{6}$  P(4) =  $\frac{1}{6}$  P(5) =  $\frac{1}{6}$  P(6) =  $\frac{1}{6}$

P(even) =  $\frac{1}{2}$  (3/6)  
P(odd) =  $\frac{1}{2}$  (3/6)

Example 5: A pair of dice is rolled. What is the probability of NOT rolling doubles?

\*There are 6 ways to roll doubles. out of 36 total  
P(doubles) =  $\frac{6}{36}$   
P(NOT doubles) =  $\frac{30}{36}$  ( $1 - \frac{6}{36}$ )

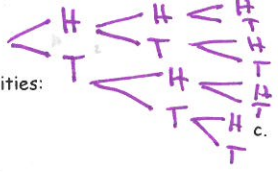
Example 6: A pair of dice are rolled. What is the probability of rolling a sum of 10 or less?  
\*What is the complement of rolling "10 or less"?

rolling > 10 = rolling 11 and 12  
 $1 - \frac{3}{36} = 1 - \frac{1}{12}$

Example 7: An experiment consists of tossing three coins.

a. List the sample space for the outcomes of the experiment.

HHH HTH THH TTH  
HHT HTT THT TTT



$2 \cdot 2 \cdot 2 = 8$  items in sample space

b. Find the following probabilities:

a. P(all heads)  $\frac{1}{8}$

c. P(no heads)  $\frac{1}{8}$

b. P(two tails)  $\frac{3}{8}$

d. P(at least one tail)  $\frac{7}{8}$

e. How could you use compliments to find d?

$P(> 1 \text{ tail}) = 1 - P(\text{all heads}) = 1 - \frac{1}{8} = \frac{7}{8}$

Example 8: A bag contains six red marbles, four blue marbles, two yellow marbles and 3 white marbles. One marble is drawn at random.

c. List the sample space for this experiment.

RRR BB Y WW  
RRR BB Y W  
15 total

d. Find the following probabilities:

a. P(red)  $\frac{6}{15}$   $\frac{2}{5}$

b. P(blue or white)  $\frac{7}{15}$

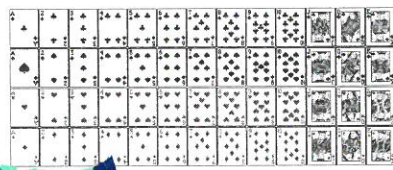
Example 9: A card is drawn at random from a standard deck of cards. Find each of the following:

e. P(heart)  $\frac{1}{4}$   $\frac{13}{52}$  Black

f. P(black card)  $\frac{1}{2}$   $\frac{26}{52}$  Black

g. P(2 or jack)  $\frac{2}{13}$   $\frac{8}{52}$  Red

h. P(not a heart)  $\frac{3}{4}$   $\frac{39}{52}$  Red  
 $= 1 - P(\heartsuit) = 1 - \frac{13}{52}$



# Unit 6 Day 2 (cont.)

Odds:

The odds of an event occurring are equal to the ratio

favorable outcomes : unfavorable outcomes

$$\text{Odds} = \# \text{ of successes} : \# \text{ of failures}$$

Ex: What are the odds, given a deck of cards 1-10, of

- a) ten  $1:9$
- b) odd  $5:5$
- c) factors of 3  $3:7$

## \*Practice\*

### Notes Sheet Ex 3-11

Example 10: What are the odds of drawing an ace at random from a standard deck of cards?

$\frac{4}{48} = \frac{1}{12}$   
 ace : not ace

Example 11: A gumball machine contains gumballs of five different colors: 36 red, 44 white, 15 blue, 20 green, and 5 orange. The machine dispenser randomly selects one gumball. What is the probability that the gumball selected is:

- 120 total gumballs
- a) Green?  $\frac{20}{120} = \frac{1}{6}$
  - b) Not green?  $\frac{100}{120}$  or  $1 - \frac{20}{120}$  or  $1 - \frac{1}{6}$
  - c) Not orange?  $\frac{115}{120}$  or  $1 - \frac{5}{120} = 1 - \frac{1}{24}$
  - d) Orange?  $\frac{5}{120}$
  - e) Not a color in the flag of the USA?  $1 - P(\text{color of USA flag}) = 1 - P(\text{red, white or blue}) = 1 - \frac{95}{120} = \frac{25}{120}$
  - f) Red, white or blue?  $\frac{95}{120} = \frac{19}{24}$   
 or  $\frac{36+44+15}{120}$

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mon denominator:

the second:

places to the left.

places to the right.

1	12
1	12
2	24
3	36
4	48
5	60
5	72
7	84
3	96
1	108
0	120
1	132
2	144