

## Interactive Student Notebook (ISNB) Details

You are a mathematician. Mathematicians use notebooks and journals to record data, illustrations, charts, graphs, and their thinking. You can also use it to record your questions and ideas. Interactive Student Notebook (ISNB), will be colorful and varied expression.

The ISNB encourages students to use their critical-thinking skills to organize information and ponder math questions. As a result, you will become a more creative independent thinker. The ISNB allows you to group notes and examples by units. Therefore you begin to see a logical flow and to understand the coherence of a unit. Your notebook serves as a chronological record of your work and helps to reinforce the major concepts in and between units.

**Materials Required:** To create **Interactive Student Notebooks**, students must bring these materials to class each day. Your Interactive notebook should only be used for MATH CLASS and COMES TO CLASS EVERYDAY. DO NOT RIP OUT OR TEAR PAGES. DO NOT WRITE YOUR HOMEWORK IN YOUR ISNB (use a separate paper or notebook). Be COLORFUL & LOVE YOUR NOTEBOOK. THIS IS YOUR STUDY GUIDE FOR THE COURSE.

1. 1 Composition Book (these are the ones that are smaller and NOT spiral bound). *Your teacher will provide ONE composition book, but it is ENCOURAGED that you bring us one in return! Please! ☺*
2. A pen, a pencil with an eraser, 2 different color markers (or colored pencils), highlighter, and glue sticks or tape.
3. Each page should have the PAGE# (You may also want to add the Unit # too).
4. **Handouts and other pages provided by your teacher MUST BE glued or taped in your ISNB.**
5. A **Table of Contents** will be kept in the front of the notebook. Write ALL entries in the Table of Contents. A template for the table of contents is located on Blackboard.
6. Your teacher will keep a master notebook in the room for your reference.
7. This **ISNB Contract will be taped into** the notebook.
8. Students can use TUTORIAL time if they wish to work on updating their ISNB or completion of activities. This is particularly valuable if there has been an absence. Staying current on all entries is vital to your success this year and your responsible for updating notebooks by seeking assistance from a peer, attending tutorials, and checking Blackboard for updates.

**STUDENT:** I understand the purpose of the Math 2 Interactive Notebook (ISNB) and will try my best to keep my notebook up-to-date, neat, organized, and complete. I will always glue or tape handouts in their appropriate location, and I will never tear pages from the notebook. I understand my notebook is a collection of my accomplishments in Math 3 class, so it is my responsibility to protect my notebook and to bring it to class **every day**. If I fall behind, I will quickly seek out notebook assistance from a peer, attend tutorial, and check Blackboard for updates.

Student Name \_\_\_\_\_ (print)

Student Signature \_\_\_\_\_ Date \_\_\_\_\_

# Factoring Bootcamp

Unit 1  
Day 1.32

Factoring: when factoring numbers or polynomials, you are finding numbers or polynomials that divide out EVENLY from the original number or polynomial.

\* We can think of factoring as "backwards distributing"

Distributing

$$2(x+3) = 2x+6$$

$$(x+1)(x-3) = x^2-2x-3$$

$$xy(xy^2+1) = x^2y^3+xy$$

Factoring

$$2x+6 = 2(x+3)$$

$$x^2-2x-3 = (x+1)(x-3)$$

$$x^2y^3+xy = xy(xy^2+1)$$

Methods of Factoring...

Flip Book !!

Always look for a GCF. Factor out the GCF.  
Examples: Factor.

$$1. 6x^2 + 9 = \underline{3(2x^2 + 3)} \underline{\hspace{2cm}}$$

$$2. 2x^3 - 4x^2 + 8x = \underline{2x(x^2 - 2x + 4)} \underline{\hspace{2cm}}$$

$$3. x^4 - x^3 = \underline{x^3(x-1)} \underline{\hspace{2cm}}$$

Greatest Common Factor

# Factoring Flip Book

"Bust" the b term (find two numbers that add to the original b and multiply to c...use these in place of the original b. Then use grouping.

Examples: Factor.

$$7. x^2 + 8x + 12 = \underline{(x+6)(x+2)}$$

$$8. x^2 + 9x - 22 = \underline{(x+11)(x-2)}$$

$$9. x^2 - 5x - 14 = \underline{(x-7)(x+2)}$$

$$10. x^2 - 4x + 3 = \underline{(x-3)(x-1)}$$

Factoring Trinomials  $x^2 + bx + c$

Group terms into pairs. Factor out a GCF (and possibly a negative), then simplify.

Examples: Factor.

$$4. x^2 - 3x - 4x + 12 = \underline{(x-4)(x-3)}$$
$$(x^2-3x) + (-4x+12)$$
$$x(x-3) - 4(x-3)$$

$$5. x^3 - 2x^2 + x - 2 = \underline{(x^2+1)(x-2)}$$

$$(x^3-2x^2) + (x-2)$$
$$x^2(x-2) + 1(x-2)$$

$$6. x^3 + 2x - 2x^2 - 4 = \underline{(x-2)(x^2+2)}$$

$$(x^3+2x) + (-2x^2-4)$$

$$x(x^2+2) - 2(x^2+2)$$

Factoring by Grouping

Rule:  
 $a^2 - b^2 = (a - b)(a + b)$

Perfect Squares:

$$\begin{aligned}2^2 &= 4 \\3^2 &= 9 \\4^2 &= 16 \\5^2 &= 25 \\6^2 &= 36 \\7^2 &= 49 \\8^2 &= 64 \\9^2 &= 81 \\10^2 &= 100 \\11^2 &= 121\end{aligned}$$

Examples: Factor.

$$15. x^2 - 16 = \underline{(x-4)(x+4)}$$

$$16. x^2 - 9 = \underline{(x-3)(x+3)}$$

$$17. 49 - x^2 = \underline{(7-x)(7+x)}$$

$$18. 4x^2 - 25 = \underline{(2x-5)(2x+5)}$$

$$19. 36 - 25x^2 = \underline{(6-5x)(6+5x)}$$

Difference of Squares  $a^2 - b^2$

"Bust" the b term (find two numbers that add to the original b and multiply to the sum of a and c... use these in place of the original b. Then use grouping.

Examples: Factor.

$$11. 3x^2 + 16x + 5 = \underline{(3x+1)(x+5)}$$

$$12. 4x^2 - 11x - 3 = \underline{(4x+1)(x-3)}$$

$$13. 2x^2 + x - 6 = \underline{(2x-3)(x+2)}$$

$$14. 8x^2 + 10x + 3 = \underline{(2x+1)(4x+3)}$$

Factoring Trinomials  $ax^2 + bx + c$



Factor completely.

25.  $2x^2 + 4x + 6 = 2(x^2 + 2x + 3)$

26.  $3x^3 + 6x^2 - 6x - 12 = (3x^2 - 6)(x + 2)$

27.  $x^2 + 12x + 36 = (x + 6)(x + 6)$

28.  $x^2 - 25 = (x + 5)(x - 5)$

29.  $64 + x^3 = (4 + x)(16 - 4x + x^2)$

30.  $4x^2 - 9 = (2x + 3)(2x - 3)$

31.  $3x^2 + 17x + 10 = (3x + 2)(x + 5)$

32.  $6x^2 + 5x - 6 = (3x - 2)(2x + 3)$

33.  $x^2 + 25 = \text{Prime !!}$

34.  $x^2 + 5x + 5 = \text{Prime !!}$

All Mixed Up Examples

Perfect  
Cubes:

$1^3 = 1$   
 $2^3 = 8$   
 $3^3 = 27$   
 $4^3 = 64$   
 $5^3 = 125$   
etc.

Rule:

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$   
 $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Examples: Factor.

20.  $x^3 + 27 = (x + 3)(x^2 - 3x + 9)$

21.  $x^3 - 27 = (x - 3)(x^2 + 3x + 9)$

22.  $8x^3 + 27 = (2x + 3)(4x^2 - 6x + 9)$

23.  $8x^3 - 125 = (2x - 5)(4x^2 + 10x + 25)$

24.  $64 - x^3 = (2 - x)(4 + 2x + x^2)$

Sum/Difference of cubes  $a^3 - b^3$  or  $a^3 + b^3$

# 1.1 Rational, Irrational, Complex Numbers

Unit 1  
Day 3

Rational: Any number that can be expressed as the quotient of two integers  
ex:  $-5, 0, 1, \frac{1}{3}, \frac{7}{9}, -\frac{4}{3}, -\frac{17}{20}, \frac{16}{13}$

Irrational: A real number that is not rational  
ex:  $\pi, e, \sqrt{2}, \sqrt{3}, \sqrt{5}, -\sqrt{6}, -\sqrt{7}$

Complex numbers involve "i".  $i = \sqrt{-1}$   
 $\hookrightarrow "a+bi"$  "i" is the imaginary number

\*There is a perfect square list in your factor flipbook\*  
Let's play with radicals!!

What makes a radical simplified?

- No  $\sqrt{\quad}$  in denominator
- No fraction under the radical
- No perfect squares under the radical.

side note:

$\sqrt{\quad}$   
this is a radical

Simplified

$$\sqrt{5}$$

$$3\sqrt{10}$$

$$2\sqrt{6}$$

$$9\sqrt{7}$$

Not Simplified

$$\sqrt{20}$$

$$3\sqrt{49}$$

$$\sqrt{\frac{1}{3}}$$

$$\sqrt{27}$$

$$\sqrt{5}/\sqrt{2}$$

Let's look at the "not simplified," and simplify.

$$\sqrt{20} = \sqrt{4 \cdot 5} = \boxed{2\sqrt{5}}$$

$$3\sqrt{49} = 3 \cdot 7 = \boxed{21}$$

$$\sqrt{27} = \sqrt{9 \cdot 3} = \boxed{3\sqrt{3}}$$

useful rule:

$$\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$$

What about  $\sqrt{\frac{1}{3}} = \frac{\sqrt{1}}{\sqrt{3}} = ?$

useful rule:

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

try this:

$$\frac{\sqrt{1}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{9}} = \boxed{\frac{\sqrt{3}}{3}}$$

↑  
"fancy 1"

\* Factor trees are useful \*

$$\begin{aligned} \sqrt{2016} &= \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 7} \\ &= \sqrt{4 \cdot 4 \cdot 9 \cdot 2 \cdot 7} \\ &= 2 \cdot 2 \cdot 3 \cdot \sqrt{14} \\ &= \boxed{12\sqrt{14}} \end{aligned}$$

Factor tree for 2016:

```

    2016
     ^
    4 504
   ^  ^
  (2)(2) 4 126
         ^  ^
        (2)(2) (2) 63
              ^  ^
             (7) 9
                ^  ^
               (3)(3)
  
```



What about  $\sqrt{-1}$ ?

There's a rule ☺  $\sqrt{-1} = i$

\*any time you see  
 $\sqrt{-a}$ , rewrite as  $i\sqrt{a}$ ,  
then simplify.

↑  
this is called  
the imaginary  
number "i"

$$\text{ex: } \sqrt{-36} = i\sqrt{36} = \boxed{6i}$$

$$\sqrt{-8} = i\sqrt{8} = i \cdot \sqrt{4} \cdot \sqrt{2} = \boxed{2i\sqrt{2}}$$

$$-\sqrt{-50} = -i\sqrt{50} = -i\sqrt{25} \cdot \sqrt{2} = \boxed{-5i\sqrt{2}}$$

$$\sqrt{5} \cdot \sqrt{-125} = \sqrt{5} \cdot i\sqrt{125} = i \cdot \sqrt{625} = \boxed{25i}$$

$$\sqrt{-10} \cdot \sqrt{-8} \neq \sqrt{80}$$

hmm...

$$\sqrt{-10} \cdot \sqrt{-8} = i\sqrt{10} \cdot i\sqrt{8} = i^2 \sqrt{80}$$

$$= i^2 \cdot \sqrt{16} \cdot \sqrt{5}$$

$$= 4i^2 \sqrt{5}$$

$$= \boxed{-4\sqrt{5}}$$

Side note...  
see if you can  
use your calculator  
to find  $i^2$ ...

Unit 1  
day 3  
1.1 cont'd

What about variables under the radical?

\*for now, let  $x \geq 0$  \*

$$\sqrt{x^7} = \sqrt{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x} = \sqrt{x \cdot x} \cdot \sqrt{x \cdot x} \cdot \sqrt{x \cdot x} \cdot \sqrt{x}$$

$$= x \cdot x \cdot x \cdot \sqrt{x}$$

$$= \boxed{x^3 \sqrt{x}}$$

or...

$$\sqrt{x^3} = \sqrt{x^2} \cdot \sqrt{x} = x \cdot \sqrt{x}$$

$$\sqrt{100x^5} = \sqrt{100} \cdot \sqrt{x^4} \cdot \sqrt{x}$$

$$= 10x^2 \cdot \sqrt{x}$$

Unit 1  
day 4

# \*Quiz Day 3 Self Study\*

Adding polynomials is just a matter of combining like terms, with some order of operations considerations thrown in. As long as you're careful with the minus signs, and don't confuse addition and multiplication, you should do fine.

There are a couple formats for adding and subtracting, and they hearken back to earlier times, when you were adding and subtracting just plain old numbers. First, you learned addition "horizontally", like this:  $6 + 3 = 9$ . You can add polynomials in the same way, grouping like terms and then simplifying.

SELF STUDY 



Adding polynomials is just a matter of combining like terms, with some order of operations considerations thrown in. As long as you're careful with the minus signs, and don't confuse addition and multiplication, you should do fine.

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- Simplify  $(2x + 5y) + (3x - 2y)$   
I'll clear the parentheses, group like terms, and then simplify:  
 $(2x + 5y) + (3x - 2y)$   
 $= 2x + 5y + 3x - 2y$   
 $= 2x + 3x + 5y - 2y$   
 $= 5x + 3y$

Horizontal addition works fine for simple examples. But when you were adding plain old numbers, you didn't generally try to add 432 and 246 horizontally; instead, you would "stack" them vertically, one on top of the other, and then add down the columns:

$$\begin{array}{r} 432 \\ + 246 \\ \hline 678 \end{array}$$

You can do the same thing with polynomials. This is how the above simplification exercise looks when it is done "vertically":

- Simplify  $(2x + 5y) + (3x - 2y)$   
I'll put each variable in its own column; in this case, the first column will be  $2x + 5y$ , the x-column, and the second column will be the y-column:  
$$\begin{array}{r} 2x + 5y \\ 3x - 2y \\ \hline 5x + 3y \end{array}$$
  
I get the same solution vertically as I got horizontally:  $5x + 3y$ .

The format you use, horizontal or vertical, is a matter of taste (unless the instructions explicitly tell you otherwise). Given a choice, you should use whichever format that you're more comfortable and successful with. Note that, for simple additions, horizontal addition (so you don't have to rewrite the problem) is probably simplest, but, once the polynomials get complicated, vertical is probably safest bet (so you don't "drop", or lose, terms and minus signs).

- Simplify  $(3x^3 + 3x^2 - 4x + 5) + (x^3 - 2x^2 + x - 4)$

I can add horizontally:

$$\begin{aligned} (3x^3 + 3x^2 - 4x + 5) + (x^3 - 2x^2 + x - 4) \\ = 3x^3 + 3x^2 - 4x + 5 + x^3 - 2x^2 + x - 4 \\ = 3x^3 + x^3 + 3x^2 - 2x^2 - 4x + x + 5 - 4 \\ = 4x^3 + 1x^2 - 3x + 1 \end{aligned}$$

...or vertically:

$$\begin{array}{r} 3x^3 + 3x^2 - 4x + 5 \\ x^3 - 2x^2 + x - 4 \\ \hline 4x^3 + 1x^2 - 3x + 1 \end{array}$$

Either way, I get the same answer:  $4x^3 + 1x^2 - 3x + 1$ .

Note that each column in the vertical addition above contains only one degree of  $x$ : the first column was the  $x^3$  column, the second column was the  $x^2$  column, the third column was the  $x$  column, and the fourth column was the constants column. This is analogous to having a thousands column, a hundreds column, a tens column, and a ones column when doing strictly-numerical addition.

- Simplify  $(7x^2 - x - 4) + (x^2 - 2x - 3) + (-2x^2 + 3x + 5)$

It's perfectly okay to have to add three or more polynomials at once. I'll just go slowly and do each step thoroughly, and it should work out right.

Adding horizontally:

$$\begin{aligned} (7x^2 - x - 4) + (x^2 - 2x - 3) + (-2x^2 + 3x + 5) \\ = 7x^2 - x - 4 + x^2 - 2x - 3 + -2x^2 + 3x + 5 \\ = 7x^2 + 1x^2 - 2x^2 - 1x - 2x + 3x - 4 - 3 + 5 \\ = 8x^2 - 2x^2 - 3x + 3x - 7 + 5 \\ = 6x^2 - 2 \end{aligned}$$

Note the 1's in the third line. Any time you have a variable without a coefficient, there is an "understood" 1 as the coefficient. If you find it helpful to write that 1 in, then do so.

Adding vertically:

$$\begin{array}{r} 7x^2 - x - 4 \\ x^2 - 2x - 3 \\ -2x^2 + 3x + 5 \\ \hline 6x^2 - 2 \end{array}$$

Either way, I get the same answer:  $6x^2 - 2$

- Simplify  $(x^3 + 5x^2 - 2x) + (x^3 + 3x - 6) + (-2x^2 + x - 2)$

Horizontally:

$$\begin{aligned} (x^3 + 5x^2 - 2x) + (x^3 + 3x - 6) + (-2x^2 + x - 2) \\ = x^3 + 5x^2 - 2x + x^3 + 3x - 6 + -2x^2 + x - 2 \\ = x^3 + x^3 + 5x^2 - 2x^2 - 2x + 3x + x - 6 - 2 \\ = 2x^3 + 3x^2 + 2x - 8 \end{aligned}$$

When you add large numbers, there are sometimes zeroes in the numbers, such as:

$$\begin{array}{r} 1002 \\ 560 \\ \hline 1562 \end{array}$$

The zeroes in "1002" stand for "zero hundreds" and "zero tens". They are what is called "placeholders", indicating that there are no hundreds or tens. If you didn't include those zeroes in the numerical expression, you'd have just in the top line "12", which isn't what you mean. The zeroes keep things lined up properly. When you vertically add polynomials that skip some of the degrees of  $x$ , you need to leave gaps, so the terms line up properly.

Vertically:

$$\begin{array}{r} x^3 + 5x^2 - 2x \\ x^3 + 3x - 6 \\ - 2x^2 + x - 2 \\ \hline 2x^3 + 3x^2 + 2x - 8 \end{array}$$

Either way, I get the same answer:  $2x^3 + 3x^2 + 2x - 8$

Subtracting polynomials works pretty much the same way....

Subtracting polynomials is quite similar to adding polynomials, but you have that pesky minus sign to deal with. Here are some examples, done both horizontally and vertically:

\*Simplify  $(x^3 + 3x^2 + 5x - 4) - (3x^3 - 8x^2 - 5x + 6)$

The first thing I have to do is take that negative through the parentheses. Some students find it helpful to put a "1" in front of the parentheses, to help them keep track of the minus sign:

Horizontally:

$$\begin{aligned} &(x^3 + 3x^2 + 5x - 4) - (3x^3 - 8x^2 - 5x + 6) \\ &= (x^3 + 3x^2 + 5x - 4) - 1(3x^3 - 8x^2 - 5x + 6) \\ &= (x^3 + 3x^2 + 5x - 4) \\ &\quad - 1(3x^3) - 1(-8x^2) - 1(-5x) - 1(6) \\ &= x^3 + 3x^2 + 5x - 4 - 3x^3 + 8x^2 + 5x - 6 \\ &= x^3 - 3x^3 + 3x^2 + 8x^2 + 5x + 5x - 4 - 6 \\ &= -2x^3 + 11x^2 + 10x - 10 \end{aligned}$$

In the horizontal case, you may have noticed that running the negative through the parentheses changed the sign on each term inside the parentheses. The shortcut here is to not bother writing in the subtraction sign or the parentheses; instead, you just change all the signs in the second row.

I'll change all the signs in the second row (shown in red below), and add down:

$$\begin{array}{r} x^3 + 3x^2 + 5x - 4 \\ -3x^3 + 8x^2 + 5x - 6 \\ \hline -2x^3 + 11x^2 + 10x - 10 \end{array}$$

Either way, I get the answer:  $-2x^3 + 11x^2 + 10x - 10$

\*Simplify  $(6x^3 - 2x^2 + 8x) - (4x^3 - 11x + 10)$

Horizontally:

$$\begin{aligned} &(6x^3 - 2x^2 + 8x) - (4x^3 - 11x + 10) \\ &= (6x^3 - 2x^2 + 8x) - 1(4x^3 - 11x + 10) \\ &= (6x^3 - 2x^2 + 8x) - 1(4x^3) - 1(-11x) - 1(10) \\ &= 6x^3 - 2x^2 + 8x - 4x^3 + 11x - 10 \\ &= 6x^3 - 4x^3 - 2x^2 + 8x + 11x - 10 \\ &= 2x^3 - 2x^2 + 19x - 10 \end{aligned}$$

Either way, I get the answer:  $2x^3 - 2x^2 + 19x - 10$

Vertically:

$$\begin{array}{r} x^3 + 3x^2 + 5x - 4 \\ - (3x^3 - 8x^2 - 5x + 6) \\ \hline \end{array}$$

Vertically, I'll write out the polynomials, leaving gaps as necessary:

$$\begin{array}{r} 6x^3 - 2x^2 + 8x \\ 4x^3 \phantom{- 2x^2} + 8x \\ \hline \end{array}$$

Then I'll change the signs in the second line, and add:

$$\begin{array}{r} 6x^3 - 2x^2 + 8x \\ -4x^3 \phantom{- 2x^2} + 11x - 10 \\ \hline 2x^3 - 2x^2 + 19x - 10 \end{array}$$



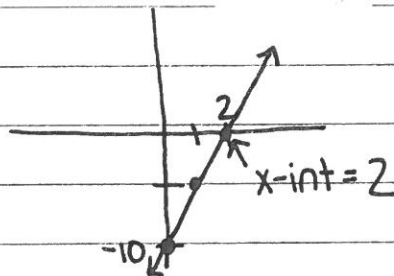
### 1.3 Solving Quadratics

Solve:  $5x - 10 = 0$

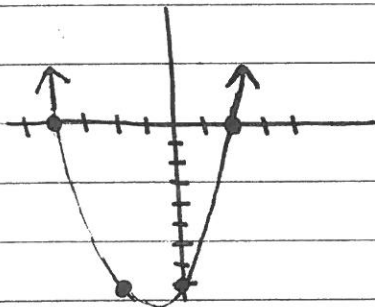
$$5x = 10$$

$$x = 2$$

Graph:  $y = 5x - 10$



Graph  $y = (x-2)(x+4)$ . x-int =  $(-4, 0)$  and  $(2, 0)$



Solve  $(x-2)(x+4) = 0$

$$x = 2 \quad x = -4$$

hmmm... same  
as x-intercepts!!

A quadratic equation is an equation that can be written in the form  $ax^2 + bx + c = 0$  with  $a \neq 0$ .

↑  
this is the standard form !!

We have many methods to solve quadratics.

→ factoring, graphing, square roots, complete the square, quad. form

Side Note...

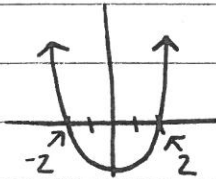
\* A solved quadratic equation can have 0, 1, or 2 solutions. \*

1.3  
Unit 1  
Day 5

I. Solving by graphing: graph  $y = ax^2 + bx + c$   
 $y = 0$   
use calculator to find intersection(s)

ex:  $x^2 - 4 = 0$

in calc.  $\begin{cases} y = x^2 - 4 \\ y = 0 \end{cases}$

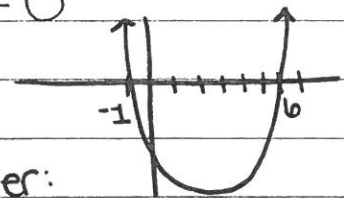


answer:  $x = 2, -2$

ex:  $x^2 - 6x = 7$

\* get into form  $ax^2 + bx + c$

in calc.  $\begin{cases} y = x^2 - 6x - 7 \\ y = 0 \end{cases}$

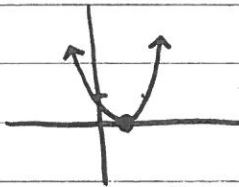


answer:

$x = -1, 6$

ex:  $x^2 - 2x + 1 = 0$

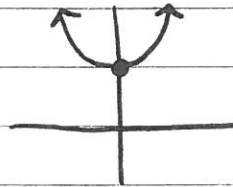
in calc.  $\begin{cases} y = x^2 - 2x + 1 \\ y = 0 \end{cases}$



answer:  $x = 1$

ex:  $x^2 + 4 = 0$

in calc.  $\begin{cases} y = x^2 + 4 \\ y = 0 \end{cases}$



answer:

no solution

## II. Solving by Square Roots

You can solve equations of the form  $x^2 = a$  by finding the square roots of each side.

\* Since  $(b)^2 = 3b$

and  $(-b)^2 = 3b$ ,

$b$  and  $-b$  are BOTH solutions to

$$x^2 = 3b.$$

ex.  $x^2 = 9$   
 $x = \pm \sqrt{9}$   
 $x = \pm 3$

ex.  $x^2 = 5$   
 $x = \pm \sqrt{5}$

ex.  $x^2 = 8$   
 $x = \pm \sqrt{8}$   
 $x = \pm 2\sqrt{2}$

ex.  $64x^2 = 4$   
 $x^2 = \frac{1}{16}$   
 $x = \pm \sqrt{\frac{1}{16}}$   
 $x = \pm \frac{1}{4}$

ex.  $2x^2 - 8 = 0$   
 $2x^2 = 8$   
 $x^2 = 4$   
 $x = \pm 2$

Fun: ex.  $x^2 + 25 = 0$   
 $x^2 = -25$   
 $x = \pm \sqrt{-25}$   
 $x = \pm 5i$

↑  
imaginary  
☺

### III. Solving by Factoring

#### \* Zero Product Property \*

For every real number a and b,  
if  $a \cdot b = 0$ , then  $a = 0$  or  $b = 0$ .

ex:  $(x+3)(2x-5) = 0$  ← this is already factored  
 $x+3=0$   $2x-5=0$  Set each factor = 0  
 $x = -3, x = 5/2$  and solve.

ex:  $x^2 - 7x + 12 = 0$  ← this is not factored...  
 $(x-3)(x-4) = 0$  so factor, then set  
 $x-3=0$   $x-4=0$  each piece = 0.  
 $x = 3, x = 4$

ex:  $x^2 + 7x = 30$  ← this is not = to zero...  
 $x^2 + 7x - 30 = 0$  ← subtract to get = 0.  
 $(x+10)(x-3) = 0$  ← factor  
 $x+10=0$   $x-3=0$  ← set each = 0  
 $x = -10, x = 3$

ex:  $4x^2 + 13x + 9 = 0$   
 $(4x+9)(x+1) = 0$   
 $x = -9/4, x = -1$

# 1.3 cont'd → Solving Quadratics

1.3  
Unit 1  
Day 6

## IV. Solving with the Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{if } ax^2 + bx + c = 0 \text{ and } a \neq 0$$

ex:  $3x^2 + 8x + 5 = 0$

$$x = \frac{-8 \pm \sqrt{8^2 - 4 \cdot 3 \cdot 5}}{2 \cdot 3} = \frac{-8 \pm \sqrt{64 - 60}}{6}$$

$$x = \frac{-8 \pm \sqrt{4}}{6} = \frac{-8 \pm 2}{6} = \frac{-8+2}{6} \text{ or } \frac{-8-2}{6}$$

answer:  $x = -\frac{6}{6}$  or  $x = -\frac{10}{6}$

$x = -1, -\frac{5}{3}$  ← 2 Real, Rational Answers

ex:  $x^2 + 2x - 13 = 0$

$$x = \frac{-2 \pm \sqrt{4 - 4 \cdot 1 \cdot (-13)}}{2 \cdot 1} = \frac{-2 \pm \sqrt{4 + 52}}{2}$$

$$x = \frac{-2 \pm \sqrt{56}}{2} = \frac{-2 \pm 2\sqrt{14}}{2}$$

$x = -1 + \sqrt{14}, -1 - \sqrt{14}$  ← 2 Irrational Answers



1.3  
Unit 1  
Day 6

ex:  $4x^2 - 12x + 9 = 0$

$$x = \frac{+12 \pm \sqrt{(-12)^2 - 4 \cdot 4 \cdot 9}}{2 \cdot 4} = \frac{12 \pm \sqrt{144 - 144}}{8}$$

$$x = \frac{12 \pm 0}{8}$$

$$x = \frac{12}{8}$$

$$x = \frac{3}{2}$$

1  
Real, Rational  
Answer

ex:  $3x^2 - 4x = -2$

$$3x^2 - 4x + 2 = 0$$

$$x = \frac{+4 \pm \sqrt{(-4)^2 - 4 \cdot 3 \cdot 2}}{2 \cdot 3} = \frac{4 \pm \sqrt{16 - 24}}{6}$$

$$x = \frac{4 \pm \sqrt{-8}}{6} = \frac{4 \pm 2i\sqrt{2}}{6}$$

$$x = \frac{2 + i\sqrt{2}}{3}, \frac{2 - i\sqrt{2}}{3}$$

2  
non-  
real  
answers

1.3 Uni  
Day 4

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$b^2 - 4ac =$$

Discriminant

<u>Discriminant</u>	<u># 3 type of answers</u>
positive perfect square	2 real, rational roots
positive non-square	2 irrational roots
zero	1 real rational root
negative number	2 imaginary roots

# 1.4 - Complete the Square

Unit 1  
Day 7

To solve standard form  $x^2 + bx + c = 0$  by "completing the square":

1. Put variable terms on left side of the equal sign and the constant term on the right:

$$x^2 + bx = -c$$

2. Take one-half the coefficient of the x-term, square it. Add this quantity to both sides:

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = -c + \left(\frac{b}{2}\right)^2$$

3. Factor the Perfect Square Trinomial on the left side of the equation and simplify the right side.  
\*\*Remember, the left always factors into  $\left(x + \frac{b}{2}\right)^2$

$$\left(x + \frac{b}{2}\right)^2 = -c + \left(\frac{b}{2}\right)^2$$

4. Take the square root of both sides:

$$\left(x + \frac{b}{2}\right) = \pm \sqrt{-c + \left(\frac{b}{2}\right)^2}$$

5. Continue to solve for x by subtracting  $\left(\frac{b}{2}\right)$  from both sides:

$$x = -\frac{b}{2} \pm \sqrt{-c + \left(\frac{b}{2}\right)^2}$$

To solve standard form  $ax^2 + bx + c = 0$  by "completing the square":

1. Put variable terms on left side of the equal sign and the constant term on the right:

$$ax^2 + bx = -c$$

2. Divide both sides by a:

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

3. Take one-half the coefficient of the x-term, square it. Add this quantity to both sides:

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

4. Factor the Perfect Square Trinomial on the left side of the equation and simplify the right side.  
\*\*Remember, the left always factors into  $\left(x + \frac{b}{2a}\right)^2$ .

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

5. Take the square root of both sides:

$$\left(x + \frac{b}{2a}\right) = \pm \sqrt{-\frac{c}{a} + \left(\frac{b}{2a}\right)^2}$$

6. Continue to solve for x by subtracting  $\left(\frac{b}{2a}\right)$  from both sides:

$$x = -\frac{b}{2a} \pm \sqrt{-\frac{c}{a} + \left(\frac{b}{2a}\right)^2}$$

Solve each using complete the square:

1.  $0 = x^2 + 6x - 12$

2.  $0 = x^2 - 6x + 12$

3.  $0 = 3x^2 + 6x - 12$

4.  $0 = 3x^2 - 6x - 3$

1.  $x^2 + 6x = +12$

side note  $(\frac{6}{2})^2 = 9$

$x^2 + 6x + 9 = +12 + 9$

$(x+3)^2 = 21$

$x+3 = \pm\sqrt{21}$

$x = -3 \pm \sqrt{21}$

2.  $x^2 - 6x = -12$

side note  $(\frac{-6}{2})^2 = 9$

$x^2 - 6x + 9 = -12 + 9$

$(x-3)^2 = -3$

$x-3 = \pm\sqrt{-3}$

$x = 3 \pm i\sqrt{3}$

3.  $3x^2 + 6x = 12$

$x^2 + 2x = 4$

$x^2 + 2x + 1 = 4 + 1$

$(x+1)^2 = 5$

$x+1 = \pm\sqrt{5}$

$x = -1 \pm \sqrt{5}$

4.  $3x^2 - 6x = 3$

$x^2 - 2x = 1$

$x^2 - 2x + 1 = 1 + 1$

$(x-1)^2 = 2$

$x-1 = \pm\sqrt{2}$

$x = 1 \pm \sqrt{2}$

# 1.4: Use Complete the Square to Rewrite Quadratic Equations.

To put standard form  $y = x^2 + bx + c$

into vertex form  $y = (x-h)^2 + k$

by "completing the square":

1. Put in standard form:	$y = x^2 + bx + c$
2. Take one-half the coefficient of the x-term, square it. Add and subtract this ("fancy zero"):	$y = \left(x^2 + bx + \left(\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c\right)$
3. Factor the Perfect Square Trinomial on the left side of the equation and simplify the right side. **Remember, the left always factors into $\left(x + \frac{b}{2}\right)^2$	$y = \left(\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c\right)$

To put standard form  $y = ax^2 + bx + c$

into vertex form  $y = a(x-h)^2 + k$

by "completing the square":

1. Put in standard form:	$y = ax^2 + bx + c$
2. Factor out "a" if $a \neq 1$ :	$y = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right)$
3. Take one-half the coefficient of the x-term, square it. Add and subtract this ("fancy zero"):	$y = a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a}\right)$
4. Factor the Perfect Square Trinomial on the left side of the equation and simplify the right side. **Remember, the left always factors into $\left(x + \frac{b}{2a}\right)^2$	$y = a\left(\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a}\right)$



Rewrite each in vertex form:

1. $y = x^2 + 6x - 12$	2. $y = x^2 - 6x + 12$
answer: $y = (x+3)^2 - 21$	answer: $y = (x-3)^2 + 3$
3. $y = 3x^2 + 6x - 12$	4. $y = 3x^2 - 6x - 3$

$$1. \quad y = x^2 + 6x + 9 - 12 - 9$$

$\swarrow$  add  $(\frac{b}{2})^2$        $\swarrow$  subtract  $(\frac{b}{2})^2$

$$y = (x+3)^2 - 21$$

$$2. \quad y = x^2 - 6x + 9 + 12 - 9$$

$$y = (x-3)^2 + 3$$

$$3. \quad y = 3x^2 + 6x - 12$$

$$y = 3(x^2 + 2x - 4)$$

$$y = 3(x^2 + 2x + 1 - 4 - 1)$$

$$y = 3((x+1)^2 - 5)$$

$$y = 3(x-1)^2 - 15$$

$$4. \quad y = 3x^2 - 6x - 3$$

$$y = 3(x^2 - 2x - 1)$$

$$y = 3(x^2 - 2x + 1 - 1 - 1)$$

$$y = 3((x-1)^2 - 2)$$

$$y = 3(x-1)^2 - 6$$