Homework Exercises
Divider/Chooser and Lone Divider methods

1. Chance and Brianna buy a pizza for $12 that is half pepperoni and half veggie. They cut the pizza into 8 slices. Chance likes veggie three times as much as pepperoni.

    a. What is the value to Chance of a slice of pepperoni?
    b. What is the value to Brianna of a slice of pepperoni?
    c. What is the value of a single slice of pepperoni?

2. Ahmed and Tiana buy a cake for $14 that is half chocolate and half vanilla. They cut the cake into 8 slices. If Ahmed likes chocolate four times as much as vanilla, what is the value of a slice that is half chocolate, half vanilla?

3. Erin, Catherine, and Shannon are dividing a large bag of candy. They randomly split the bag into three bowls. The values of the entire bag and each of the three bowls in the eyes of each of the players are shown below. For each player, identify which bowls they value as a fair share.

	Whole Bag	Bowl 1	Bowl 2	Bowl 3
Erin	$5	$2.75	$1.25	$1.00
Catherine	$4	$0.75	$2.50	$0.75
Shannon	$8	$1.75	$2.25	$4.00

4. Jenna, Tatiana, and Nina are dividing a large bag of candy. They randomly split the bag into three bowls. The values of the entire bag and each of the three bowls in the eyes of each of the players are shown below. For each player, identify which bowls they value as a fair share.

	Whole Bag	Bowl 1	Bowl 2	Bowl 3
Jenna	$8	$4.50	$0.75	$2.75
Tatiana	$4	$1.00	$1.00	$2.00
Nina	$6	$1.75	$2.50	$1.75

5. Dustin and Kendra want to split a bag of fun-sized candy, and decide to use the divider-chooser method. The bag contains 100 Snickers, 100 Milky Ways, and 100 Reese's, which Dustin values at $1, $5, and $2 respectively. (This means Dustin values the 100 Snickers together at $1, or $0.01 for 1 Snickers).
    If Kendra is the divider, and in one half puts: 25 Snickers, 20 Milky Ways, and 60 Reese's
    a. What is the value of this half in Dustin's eyes?
    b. Does Dustin consider this a fair share?
    c. If Dustin was a divider, find a possible division that is consistent with his value system.

6. Dustin and Kendra want to split a bag of fun-sized candy, and decide to use the divider-chooser method. The bag contains 100 Snickers, 100 Milky Ways, and 100 Reese's, which Dustin values at $1, $3, and $5 respectively. (This means Dustin values the 100 Snickers together at $1, or $0.01 for 1 Snickers).
   If Kendra is the divider, and in one half puts: 30 Snickers, 40 Milky Ways, and 66 Reese's

   a. What is the value of this half in Dustin's eyes?
   b. Does Dustin consider this a fair share?
   c. If Dustin was a divider, find a possible division that is consistent with his value system.

7. Maggie, Meredith, Holly, and Zoe are dividing a piece of land using the lone-divider method. The values of the four pieces of land in the eyes of the each player are shown below.

	Piece 1	Piece 2	Piece 3	Piece 4
Maggie	21%	27%	32%	20%
Meredith	27%	29%	22%	22%
Holly	23%	14%	41%	22%
Zoe	25%	25%	25%	25%

a. Who was the divider?
b. If playing honestly, what will each player's declaration be?
c. Find the final division.

8. Cody, Justin, Ahmed, and Mark are going to share a vacation property. The year will be divided into 4 time slots using the lone-divider method. The values of each time slot in the eyes of the each player are shown below.

	Time 1	Time 2	Time 3	Time 4
Cody	10%	35%	34%	21%
Justin	25%	25%	25%	25%
Ahmed	19%	24%	30%	27%
Mark	23%	31%	22%	24%

a. Who was the divider?
b. If playing honestly, what will each player's declaration be?
c. Find the final division.

Sealed Bid method

9. Four heirs (A, B, C, and D) must fairly divide an estate consisting of two items – a desk and a vanity - using the method of sealed bids. The players' bids (in dollars) are:

	A	B	C	D
Desk	320	240	300	260
Vanity	220	140	200	180

   a. What is A's fair share?
   b. Find the initial allocation.
   c. Find the final allocation.

10. Three heirs (A, B, C) must fairly divide an estate consisting of three items - a house, a car, and a coin collection - using the method of sealed bids. The players' bids (in dollars) are:

	A	B	C
House	180,000	210,000	220,000
Car	12,000	10,000	8,000
Coins	3,000	6,000	2,000

   a. What is A's fair share?
   b. Find the initial allocation.
   c. Find the final allocation.

11. As part of an inheritance, four children, Abby, Ben and Carla, are dividing four vehicles using Sealed Bids. Their bids (in thousands of dollars) for each item is shown below. Find the final allocation.

	Abby	Ben	Carla
Motorcycle	10	9	8
Car	10	11	9
Tractor	4	1	2
Boat	7	6	4

12. As part of an inheritance, four children, Abby, Ben, Carla, and Dan, are dividing four vehicles using Sealed Bids. Their bids (in thousands of dollars) for each item is shown below. Find the final allocation.

	Abby	Ben	Carla	Dan
Motorcycle	6	7	11	8
Car	8	13	10	11
Tractor	3	1	5	4
Boat	7	6	3	8

13. Amanda, Brian, and Charlene are heirs to an estate that includes a house, a boat, a car, and $150,000 in cash. Their bids (in thousands of dollars) for each item is shown below. Find the final settlements for each heir.

	House	Boat	Car
Amanda	80,000	5,000	8,000
Brian	70,000	9,000	11,000
Charlene	76,000	7,000	13,000

14. In number 13, if Amanda and Charlene had both submitted bids of $80,000 for the house, how would you resolve the tie?

15. Garfield and Marmaduke are heirs to an estate that contains only a summer cottage. Garfield bids $70,000 and Marmaduke bids $60,000.

   a. What does Garfield feel is a fair share? Marmaduke?
   b. What is the difference between Garfield's fair share and his bid for the cottage?
   c. Because the value Garfield assigned to the cottage is more than Garfield's fair share, Garfield must pay cash into the estate. How much cash must Garfield pay?
   d. Marmaduke is given an amount of cash from Garfield's payment equal to Marmaduke's fair share. How much does Marmaduke receive? If the remaining cash is divided equally, what will be the final value of Marmaduke's settlement? Of Garfield's?
   e. Garfield must borrow money in order to pay into the estate, and the interest on this loan is $2,000. Do you think this should be considered when arriving at a settlement? If so, suggest how to revise the settlement.

Last Diminisher method

16.

A 6-foot sub valued at $30 is divided among five players (P_1, P_2, P_3, P_4, P_5) using the last-diminisher method. The players play in a fixed order, with P_1 first, P_2 second, and so on. In round 1, P_1 makes the first cut and makes a claim on a piece. For each of the remaining players, the value of the *current* claimed piece at the time it is their turn is given in the following table:

	P_2	P_3	P_4	P_5
Value of the current claimed piece	$6.00	$8.00	$7.00	$6.50

a. Which player gets his or her share at the end of round 1?
b. What is the value of the share to the player receiving it?
c. How would your answer change if the values were:

	P_2	P_3	P_4	P_5
Value of the current claimed piece	$6.00	$8.00	$7.00	$4.50

17.

A huge collection of low-value baseball cards appraised at $100 is being divided by 5 kids (P_1, P_2, P_3, P_4, P_5) using the last-diminisher method. The players play in a fixed order, with P_1 first, P_2 second, and so on. In round 1, P_1 makes the first selection and makes a claim on a pile of cards. For each of the remaining players, the value of the *current* pile of cards at the time it is their turn is given in the following table:

	P_2	P_3	P_4	P_5
Value of the current pile of cards	$15.00	$22.00	$18.00	$19.00

a. Which player gets his or her share at the end of round 1?
b. What is the value of the share to the player receiving it?
c. How would your answer change if the values were:

	P_2	P_3	P_4	P_5
Value of the current pile of cards	$15.00	$22.00	$18.00	$21.00

Domain and Range

For each of the following functions, determine the domain algebraically (and confirm graphically), and determine the range graphically.

1) $f(x) = 3x^2 - 6$

2) $g(x) = \dfrac{1}{x-1}$

3) $h(x) = \sqrt{3-x}$

4) $j(x) = \dfrac{\sqrt{x}}{x-4}$

5) $k(x) = \dfrac{x^2 - 3x + 6}{x^2 - 3x - 10}$

6) $m(x) = \dfrac{2x}{\sqrt{x^2 - 9}}$

7) $n(x) = \sqrt{8x^3 - 24x^2}$

8) $p(x) = \dfrac{\sqrt{x-3}}{\sqrt{x+4}}$

Rational Expressions:

Reducing Rational Expressions

1. Reduce $\dfrac{x^2-9}{x-3}$

2. Reduce $\dfrac{y^2-5y-6}{y^2-1}$

3. Reduce $\dfrac{x^2-3x+ax-3a}{x^2-ax-3x+3a}$

4. Reduce $\dfrac{a-b}{b-a}$

5. Reduce $\dfrac{x^2-25}{5-x}$

6. Reduce $\dfrac{2x^3+2x^2-24x}{x^3+2x^2-8x}$

Multiplying and Dividing Expressions

1. Multiply $\dfrac{8x^3}{27y^8}\cdot\dfrac{9y^3}{12x^2}$

2. Multiply $\dfrac{x-3}{x^2-4}\cdot\dfrac{x+2}{x^2-6x+9}$

3. Divide $\dfrac{3x-9}{x^2-x-20}\cdot\dfrac{x^2+2x-15}{x^2-25}$

4. Multiply $\left(x^2-49\right)\left(\dfrac{x+4}{x+7}\right)$

5. Multiply $a(a+5)(a-5)\left(\dfrac{a+4}{a^2-5a}\right)$

Adding and Subtracting Expressions

1. Add $\dfrac{-2}{x^2-2x-3}+\dfrac{3}{x^2-9}$

2. Add $\dfrac{x}{x^2-1}+\dfrac{1}{x^2-1}$

3. Subtract $\dfrac{x+4}{2x+10}-\dfrac{5}{x^2-25}$

4. Add $\dfrac{5}{x}+\dfrac{3}{x}$

Rational expressions: given the following f(x) expressions, find

a. $f(x+a)$

b. $f(a)$

c. $\dfrac{f(x+a)-f(a)}{x}$ (be sure to note any domain restrictions)

1. $f(x)=3x$

2. $f(x)=3x+9$

3. $f(x)=3x^2$

4. $f(x)=x^3$

5. $f(x)=x^{\frac{1}{2}}$

6. $f(x)=3x^2+5x-6$

Challenge

7. $f(x)=\sin(x)$

$$\begin{aligned}
\sin(\alpha+\beta) &= \sin\alpha\,\cos\beta + \cos\alpha\,\sin\beta \\
\cos(\alpha+\beta) &= \cos\alpha\,\cos\beta - \sin\alpha\,\sin\beta \\
\sin(\alpha-\beta) &= \sin\alpha\,\cos\beta - \cos\alpha\,\sin\beta \\
\cos(\alpha-\beta) &= \cos\alpha\,\cos\beta + \sin\alpha\,\sin\beta
\end{aligned}$$