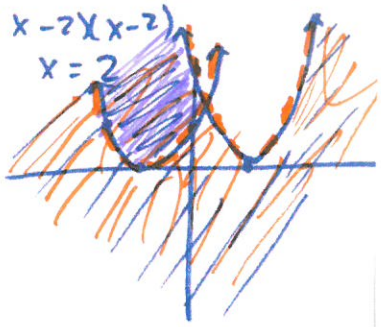
Honors Math 2
Midterm Mastery Review

Name KEY

1. On a separate sheet of graph paper, graph $y > x^2 + 2x + 1$ and $y < x^2 - 4x + 4$ and find the intersection.

$(x+1)(x+1)$
$x = -1$

$(x-2)(x-2)$
$x = 2$



2. Solve: $\sqrt{x+2}^2 = x^2$

$x + 2 = x^2$
$0 = x^2 - x - 2$
$0 = (x-2)(x+1)$ $\boxed{x = 2}$ or $\cancel{-1}$

3. Solve $(x+2)^{3/4} + 3 = 30$

$-3 \quad -3$
$(x+2)^{3/4 \cdot 4/3} = 27^{4/3}$
$x + 2 = 81$
$\boxed{x = 79}$

4. Solve: $\sqrt[3]{3x+1} + 10 = 5$

$\sqrt[3]{3x+1}^3 = -5^3$
$3x + 1 = -125$
$3x = -126$
$\boxed{x = -42}$

5. Simplify $\left(\frac{\sqrt{b}}{\sqrt[4]{a^3}}\right)^{-8} = \left(\frac{b^{1/2}}{a^{3/4}}\right)^{-8} = \frac{b^{1/2 \cdot -8}}{a^{3/4 \cdot -8}} = \frac{b^{-4}}{a^{-6}}$ $\boxed{\frac{a^6}{b^4}}$

6. Solve $3x^3 = 48x$

$3x^3 - 48x = 0$
$3x(x^2 - 16) = 0$
$3x(x+4)(x-4) = 0$

$3x = 0 \quad x+4 = 0 \quad x-4 = 0$
$\boxed{x = 0, \; x = -4, \; x = 4}$

7. Explain how the function has changed from the parent graph. $f(x) = -\sqrt{x+8} - 5$

Reflected over x-axis, translated left 8 and down 5

8. Write an equation for the translation of $y = \frac{5}{x}$ that has the asymptotes x = -2 and y = -8.

$\boxed{y = \frac{5}{x+2} - 8}$

9. Simplify $\sqrt[7]{x^3} \cdot \sqrt[14]{x^5}$

$x^{3/7} \cdot x^{5/14} = x^{3/7 + 5/14} = x^{6/14 + 5/14}$ $\boxed{x^{11/14}}$ or $\sqrt[14]{x^{11}}$

10. Simplify: $\sqrt[4]{10x^7y^3} \cdot \sqrt[4]{60xy^8} = x\sqrt[4]{10x^3y^3} \cdot y^2\sqrt[4]{60x}$

$xy^2\sqrt[4]{600x^4y^3}$ → $x \cdot x\, y^2\sqrt[4]{600y^3}$

$\boxed{x^2y^2\sqrt[4]{600y^3}}$

11. Describe how the parabola $y = -(x - 4)^2 + 3$ is shifted from $y = x^2$.

reflect over x-axis, right 4 up 3

12. Sketch the graph of the function on a separate piece of paper. $y = x^2 + 15x + 54$

a. *Find the x-intercepts.* (-9, 0) (-6, 0)

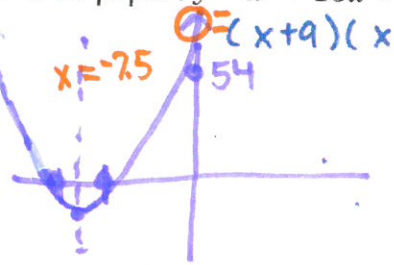
b. *Find the axis of symmetry.* $x = -7.5$ → $x = \frac{-b}{2a}$

c. *Find the vertex.* (-7.5, -2.25)

d. *Find the y-intercept.* (0, 54)

e. *Is the vertex a max or a min?* min

$0 = (x+9)(x+6)$



13. Find the equation of a quadratic function with intercepts at (-2, 0) and (4, 0) and a vertex at (1, 6). (x, y)

$y = a(x - root_1)(x - root_2)$ $root_1$ $root_2$

$y = a(x+2)(x-4)$

$6 = a(1+2)(1-4)$

$6 = a(-9)$

$-6/9 = a$ $a = -2/3$

$y = -2/3 (x+2)(x-4)$

$y = -2/3 (x^2 - 2x - 8)$

$y = -\frac{2}{3}x^2 + \frac{4}{3}x + \frac{16}{3}$

Factor and find the **solutions.**

14. $2v^2 + 11v + 5 = 0$

$2 \cdot 5 = 10$

$10 + 1 = 11$

$(2v^2 + 10v) + (v + 5) = 0$

$2v(v+5) + 1(v+5) = 0$ → $(2v+1)(v+5) = 0$

$2v + 1 = 0$

$2v = -1$

$v = -1/2$

$v = -5$

15. $7a^2 + 53a + 28 = 0$

$(7a^2 + 4a) + (49a + 28) = 0$

$a(7a+4) + 7(7a+4) = 0$ → $(a+7)(7a+4)$

$7a + 4 = 0$

$7a = -4$

$a = -4/7$

$a = -7$

16. $16b^2 + 60b - 100 = 0$

$4(4b^2 + 15b - 25) = 0$

$4(4b^2 + 20b - 5b - 25) = 0$

$4(4b(b+5) - 5(b+5)) = 0$ → $4(4b-5)(b+5) = 0$

$4b - 5 = 0$

$4b = 5$

$b = 5/4$

$b = -5$

Find the **discriminant** and tell the number/type of solutions.

17. $b^2 + 16b + 64 = 0$

$16^2 - (4 \cdot 1 \cdot 64) = 0$

$b^2 - 4ac$

1 real rational solution

18. $x^2 - 4x + 24 = 0$

$(-4)^2 - (4 \cdot 1 \cdot 24) = -80$

2 imaginary solutions

19. $2k^2 + 22k + 60 = 0$

$2(k^2 + 11k + 30)$ $(11)^2 - (4 \cdot 1 \cdot 30) = 1$

2 real rational solutions

20. The following function models how much money a certain company makes after a certain amount of time. At what time did they make the least amount of money?

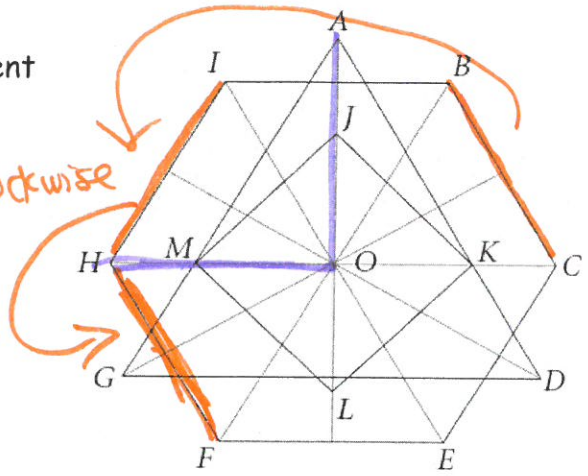
$v(t) = 800 - 28t + .25t^2$ $t = 56$

↳ find x-value of the minimum (vertex)

use $x = \frac{-b}{2a}$ OR calculator

The large triangle, quadrilateral, and hexagon are all regular polygons. Find the image of each point or segment for the given rotation. (Hint: the Green segments form 30 degree angles).

* standard direction is counter clockwise



21. 120° rotation of B about O — H

22. 270° rotation of L about O — M

23. 60° rotation of E about O — C

24. 300° rotation of $\overline{IB}$ about O — BC

25. 240° rotation of G about O — A

26. 180° rotation of $\overline{JK}$ about O — LM

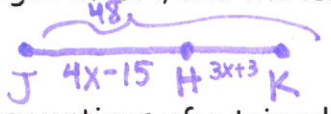
* 27. a. Find the image of the composition of reflecting segment BC over line AO, then again over line HO.

IH HF

b. Describe the single transformation that could accomplish the composition from part a.

Reflection over $\overleftrightarrow{IE}$ OR Rotation 180° (double reflection over intersecting lines) ← rotation

28. Given that H is between J and K, JK = 48, JH = 4x - 15, and HK = 3x + 3, find the value of x, the length of JH, and the length of HK.



$4x-15+3x+3=48$

$7x-12=48$

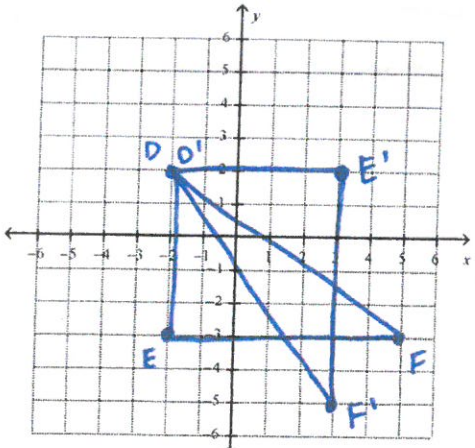
$7x=60$

$x=\frac{60}{7}$

$JH=4\left(\frac{60}{7}\right)-15=\frac{135}{7}$

$HK=3\left(\frac{60}{7}\right)+3=\frac{201}{7}$

29. The vertices of a triangle are D(-2, 2), E(-2, -3) and F(5, -3). Graph and label the image with a reflection over the line y = –x. Name the image vertices below.

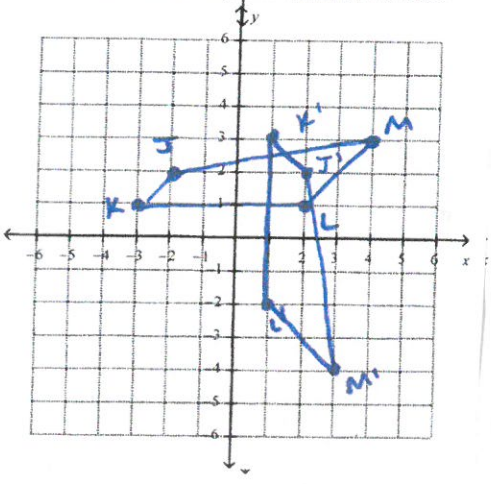


D' (-2, 2) E' (3, 2) F' (3, -5)

Write the algebraic rule for a reflection over y = –x.

(x, y) → (-y, -x)

30. Graph and label the points J(-2, 2), K(-3, 1), L(2, 1) and M(4, 3) and then rotate the figure 270°. Graph and label the image points, and write their coordinates below. Then, write the algebraic rule for the transformation.



J' (2, 2) K' (1, 3)

L' (1, -2) M' (3, -4)

Write the algebraic rule for the rotation 270°:

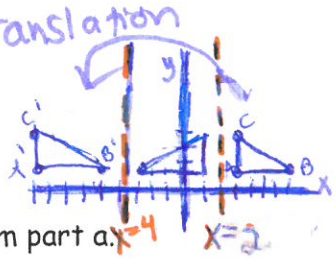
(x, y) → (y, -x)

31. Given A(3, 1), B(7, 1), C(3, 4)...

a. Find the composition of reflecting $\triangle ABC$ over x = 2, then over x = -4.

double reflection across parallel lines = translation

image of the $A'(-9,1)\ B'(-5,1)\ C'(-9,4)$



b. Describe the single transformation that could accomplish the composition from part a.

Translation left 12

32. Graph $\triangle ABC$ with A(5, -3), B(2, -4), and C(1, 1), then graph the image of $\triangle ABC$ after the translation of < 4, 1 > then reflection over the x-axis.

Label all your points then, write the coordinates of the final image below.

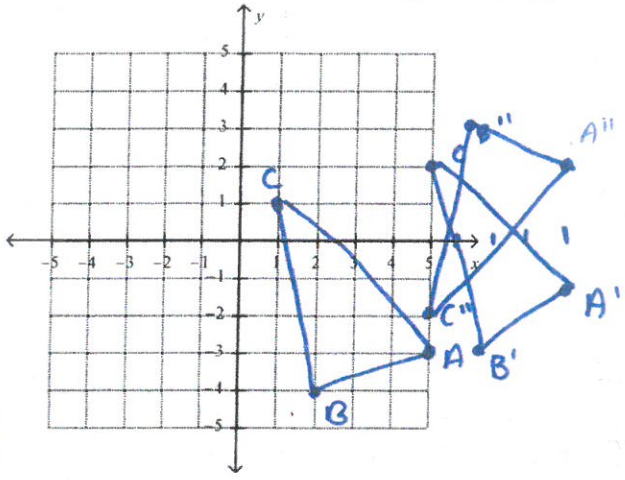


Image $A'(9,-2)\ B'(6,-3)\ C'(5,2)$ $A''(9,2)\ B''(6,3)\ C''(5,-2)$

Write the Algebraic Rule: $(x,y)\to(x+4,-y-1)$

$(x,y)\to(x+4,y+1)$ then $(x,y)\to(x,-y)$

33. Given A is the midpoint of segment SM. If $SA = x^2 + 3x$ and $AM = x + 35$, find SA.

$x^2+3x = x+35$

$x^2+2x-35=0$

$(x+7)(x-5)$

$x=-7,5$

with $x=-7$: $SA=(-7)^2+3(-7)$, $49-21$

$SA=28$ or 40

with $x=5$: $SA=(5)^2+3(5)$, $25+15$

$SA=40$

34. ~~Find~~ Simplify $3(x+2)^2-(x+2)-4(5x+1)$

$3(x+2)(x+2)-(x+2)-4(5x+1)$

$3(x^2+4x+4)-x-2-20x-4$

$3x^2+12x+12-21x-6$

$3x^2-9x+6$

35. Find the vertex form of $y = 2x^2 + 6x + 7$. Show your work by hand.

use to check in calc. $y=a(x-h)^2+k$, vertex (h,k)

$y=2(x^2+3x+\frac{9}{4}-\frac{9}{4}+\frac{7}{2})$

$y=2((x+\frac{3}{2})^2+5/4)$

$y=2(x+\frac{3}{2})^2+\frac{5}{2}$

36. Solve the inequality and write your answer in set notation. $0 \geq 3x^2 - 2x - 5$

$0=(3x-5)(x+1)$

$x=5/3, -1$

$\{x \mid -1 \leq x \leq 5/3\}$

test • $x=-2 \to 0 \geq 3(-2)^2-2(-2)-5$ false! ∞ shade there

• $x=0 \to 0 \geq 3(0)^2-2(0)-5$ true → shade there

• $x=2 \to 0 \geq 3(2)^2-2(2)-5$ false!

37. Solve for x. $0 = x^2 - 4x - 8$

$4+8=x^2-4x+4$ $(-2)^2=4$

$12=(x-2)^2$

$\pm\sqrt{12}=x-2$

$x=2\pm\sqrt{12}$

$x=2\pm2\sqrt{3}$

38. Solve $\sqrt{3x+7}=x-1$ → $(x-1)^2$

$(x-1)(x-1)$

$3x+7=x^2-2x+1$

$0=x^2-5x-6$

$0=(x-6)(x+1)$

$x=6, -1$

check $\sqrt{3\cdot6+7}\stackrel{?}{=}6-1$, $\sqrt{25}=5$ ✓

check $\sqrt{3(-1)+7}\stackrel{?}{=}-1-1$, $\sqrt{4}=-2$ ✗

don't shade there!

39. For each function, find the requested information. Graph on separate paper, using at least 4 points.

a. $f(x)=\dfrac{3}{x+2}-4$

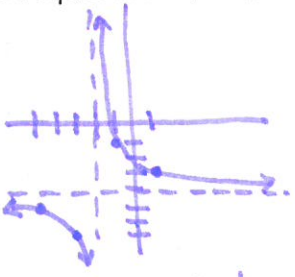
Domain: $(-\infty,-2)\cup(-2,\infty)$

Range: $(-\infty,-4)\cup(-4,\infty)$

Asymptotes: $x=-2,\ y=-4$

Explain how graph changed from parent: vertical stretch by 3, translated left 2 + down 4

(parent $y=\frac{1}{x}$)



b. $g(x)=\sqrt[3]{x-2}+4$

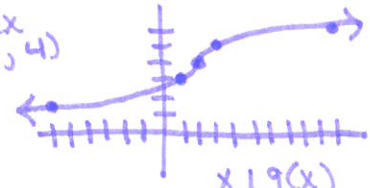
vertex opp., same (2, 4)

Domain: $(-\infty,\infty)$

Range: $(-\infty,\infty)$

Vertex: $(2,4)$

Explain how graph changed from parent: translated right 2 and up 4



x	g(x)
-6	2
1	3
2	4 vertex
3	5
10	6

40. If x varies directly as the cube root of y, and x = 6 when y = 27, find x when y = 64.

$x=k\sqrt[3]{y}$

$6=k\sqrt[3]{27}$

$6=k\cdot3$

$k=2$

1st use to make equation $x=2\sqrt[3]{y}$

2nd AFTER have equation $x=2\sqrt[3]{64}$ $x=8$