Unit 4
Day 7

Inverse Variation

Review:

Direct Variation

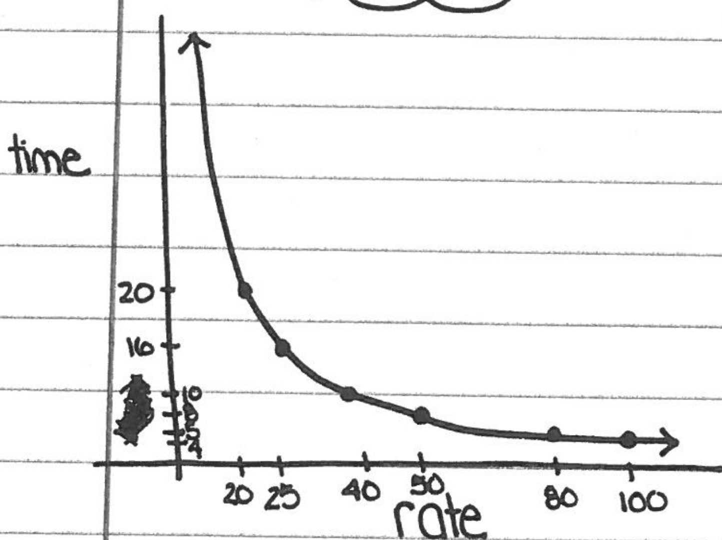
$y = kx$ (example: I earn \$9 per hour
salary = $9 \cdot$ hours)

k is a non-zero number.
k = "constant of variation"

New: Inverse Variation
$y = k/x$
or $xy = k$

example: distance = rate $\cdot$ time
- the distance to Atlanta is 400 miles. Fill in the chart:

rate	time
100mph	4 hr
80mph	5hr
50mph	8 hr
40mph	10hr
25mph	16hr
20mph	20hr



As rate ↑, time ↓.

Ex: A cookie dough recipe makes 120 in^3 of dough.
I am making a rectangular cookie pizza.
The surface of the pizza is inversely proportional to the depth of the pizza. Write a model:
$120 = s \cdot d$
My cookie pizza pan has a depth of $1/2$ inch.
What is the surface of my pizza? ($240 in^2$)

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Suppose that x and y vary inversely. Write a function that models each inverse variation.

1. $x = 1$ when $y = 11$
2. $x = -13$ when $y = 100$
3. $x = 1$ when $y = 1$
4. $x = 28$ when $y = -2$
5. $x = 1.2$ when $y = 3$
6. $x = 2.5$ when $y = 100$

Is the relationship between the values in each table a direct variation, an inverse variation, or neither? Write equations to model the direct and inverse variations.

7.

x	3	8	10	22
y	15	40	50	110

8.

x	3	5	7	10.5
y	14	8.4	6	4

9.

x	0.5	2.1	3.5	11
y	1	4.2	7	22

10.

x	0.1	3	6	24
y	3	0.1	0.05	0.0125

11.

x	7	3	1	$\frac{1}{5}$
y	$\frac{1}{7}$	$\frac{1}{3}$	1	5

12.

x	10	12	20	23
y	2	$2\frac{2}{5}$	4	$5\frac{3}{5}$

Suppose that x and y vary inversely. Write a function that models each inverse variation and find y when $x = 10$.

13. $x = 20$ when $y = 5$
14. $x = 20$ when $y = -4$
15. $x = 5$ when $y = -\frac{1}{3}$

1) $xy = 11$
$y = 11/x$

2) $xy = -130$
$y = -130/x$

3) $xy = 1$
$y = 1/x$

4) $xy = -56$
$y = -56/x$

5) $xy = 3.6$
$y = 3.6/x$

6) $xy = 25$
$y = 25/x$

7) $xy \neq k$
$y = 5x$
direct

8) $xy = 42$
inverse

9) $xy \neq k$
$y = 2x$
direct

10) $xy = 0.3$
inverse

11) $xy = 1$
inverse

12) $xy \neq k$
$y = \frac{1}{5}x$
direct

13) $xy = 100$
$10 \cdot y = 100$
$\boxed{y = 10}$

14) $xy = -80$
$10 \cdot y = -80$
$\boxed{y = -8}$

15) $xy = -\frac{5}{3}$
$10y = -\frac{5}{3}$
$\boxed{y = -\frac{1}{6}}$

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Examples of Combined Variations

Combined Variation	Equation Form
y varies directly with the square of x.	$y = kx^2$
y varies inversely with the cube of x.	$y = \frac{k}{x^3}$
z varies jointly with x and y.	$z = kxy$
z varies jointly with x and y and inversely with w.	$z = \frac{kxy}{w}$
z varies directly with x and inversely with the product of w and y.	$z = \frac{kx}{wy}$

Describe the combined variation that is modeled by each formula.

16. $A = \pi r^2$ **17.** $A = 0.5bh$ **18.** $h = \frac{2A}{b}$ **19.** $V = \frac{Bh}{3}$

20. $V = \pi r^2 h$ **21.** $h = \frac{V}{\pi r^2}$ **22.** $V = \ell wh$ **23.** $\ell = \frac{V}{wh}$

Write the function that models each relationship. Find z when $x = 4$ and $y = 9$.

24. z varies directly with x and inversely with y. When $x = 6$ and $y = 2$, $z = 15$.

25. z varies jointly with x and y. When $x = 2$ and $y = 3$, $z = 60$.

26. z varies directly with the square of x and inversely with y. When $x = 2$ and $y = 4$, $z = 3$.

27. z varies inversely with the product of x and y. When $x = 2$ and $y = 4$, $z = 0.5$.

16. Area is directly proportional to radius squared; $k = \pi$

21. height is directly proportional to Volume and inversely proportional to radius squared; $k = \frac{1}{\pi}$

24. $z = \frac{kx}{y}$ $\quad 15 = \frac{k \cdot 6}{2}$ $\quad 15 = k \cdot 3$ $\quad 5 = k$ $\quad z = \frac{5 \cdot 4}{9} = \boxed{\frac{20}{9}}$

25. $z = k \cdot xy$ $\quad 60 = k \cdot 2 \cdot 3$ $\quad 60 = 6k$ $\quad 10 = k$ $\quad z = 10 \cdot 4 \cdot 9 = \boxed{360}$

26. $z = \frac{k \cdot x^2}{y}$ $\quad 3 = \frac{k \cdot 2^2}{4}$ $\quad 3 = \frac{k \cdot 4}{4}$ $\quad k = 1$ $\quad z = \frac{1 \cdot 16}{9} = \boxed{\frac{16}{9}}$

27. $z = \frac{k}{xy}$ $\quad 0.5 = \frac{k}{2 \cdot 4}$ $\quad \frac{1}{2} = \frac{k}{8}$ $\quad k = 4$ $\quad z = \frac{4}{4 \cdot 9}$ $\quad z = \boxed{\frac{1}{9}}$

Unit 3
Day 7

52. Construction A concrete supplier sells premixed concrete in 300-ft^3 truckloads. The area A that the concrete will cover is inversely proportional to the depth d of the concrete.

a. Write a model for the relationship between the area and the depth of a truckload of poured concrete.

b. What area will the concrete cover if it is poured to a depth of 0.5 ft? A depth of 1 ft? A depth of 1.5 ft?

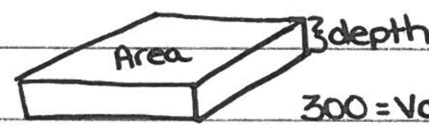
c. When the concrete is poured into a circular area, the depth of the concrete is inversely proportional to the square of the radius r. Write a model for this relationship.

56. Writing Explain why 0 cannot be in the domain of an inverse variation.

57. Critical Thinking Suppose that (x_1, y_1) and (x_2, y_2) are values from an inverse variation. Show that $\frac{x_1}{x_2} = \frac{y_2}{y_1}$.

58. Open-Ended The height h of a cylinder varies directly with its volume V and inversely with the square of its radius r. Find at least four ways to change the volume and radius of a cylinder so that its height is quadrupled.

52. $A = \frac{k}{d}$ a. $A \cdot d = 300$



300 = Volume

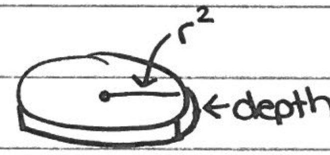
b. $d = 0.5$ft $A = 600$ft^2

$d = 1$ft $A = 300$ft^2

$d = 1.5$ $A = 200$ft^2

c. $d = \frac{k}{r^2}$

$300/\pi = d \cdot r^2$



$V = \pi r^2 \cdot d$

$\frac{V}{\pi} = \frac{300}{\pi} =$ "k"

56. $xy = k$

if $x = 0$, xy could only $= 0$ $\therefore$ $k \neq 0$

57. $x_1 y_1 = k$ and $x_2 y_2 = k$

so, $x_1 y_1 = x_2 y_2$

and, $\frac{x_1}{x_2} = \frac{y_2}{y_1}$

58. $h = \frac{kV}{r^2}$

* mult V by 4

* mult r by $\frac{1}{2}$

* mult V by 16 and r by 2

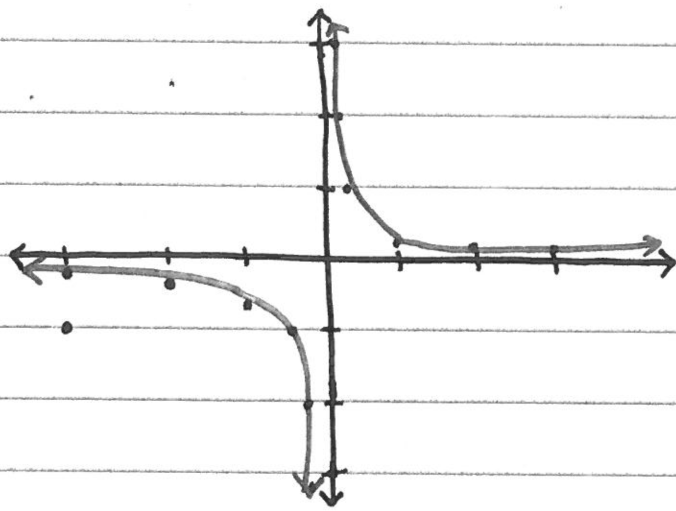
Graphing Inverse Variation

Unit 4
Day 8

$y = 3/x$

x	-9	-6	-3	-1	-1/2	-1/3	1/3	1/2	1	3	6	9
y												

* what happens when $x = 0$?



domain: $x \in \mathbb{R} \neq 0$
range: $y \in \mathbb{R} \neq 0$

The graph has 2 parts – "branches."
The x-axis is a horizontal asymptote.
The y-axis is a vertical asymptote.

In this graph, we define asymptote: a line our graph approaches, but doesn't touch

Translation:

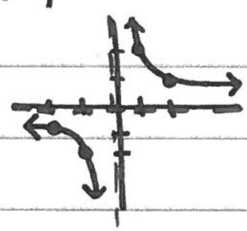
$y = \frac{k}{x-b} + c$

b: shift right/left
c: shift up/down
k: vertical stretch
$k < 0$: reflect over x-axis

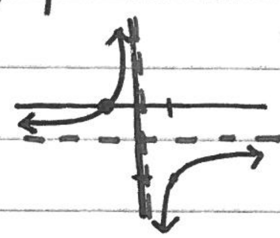
Unit 4
Day 8

Sketch a graph of the following. Use a colored pencil or marker to make a dotted asymptote.

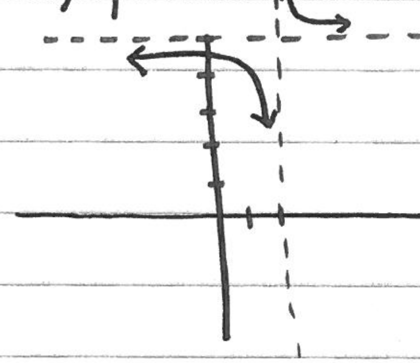
1) $y = 2/x$



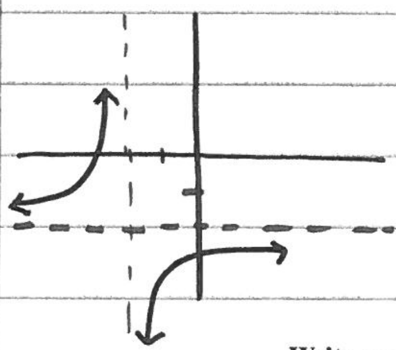
2) $y = \frac{-1}{x} - 1$



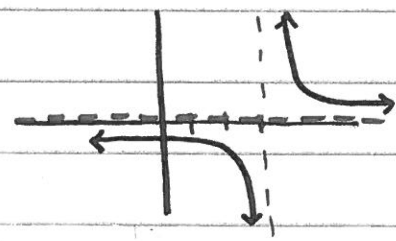
3) $y = \frac{1}{x-2} + 5$



4) $y = \frac{-2}{x+2} - 2$



5) $y = \frac{1}{x-3}$



Write an equation for the translation of $y = \frac{2}{x}$ that has the given asymptotes.

22. $x = 0$ and $y = 4$ **23.** $x = -2$ and $y = 3$ **24.** $x = 4$ and $y = -8$

25. a. Budgeting A high school spends $750 each year on student academic achievement awards. The amount spent per award depends on how many awards are given. Write and graph a function of the number *a* of awards given and the cost *c* of each award. Find the asymptotes.

b. Explain how the asymptotes are related to the given facts.

22) $y = \frac{2}{x} + 4$

23) $y = \frac{2}{x+2} + 3$

24) $y = \frac{2}{x-4} - 8$

25) $750 = a \cdot c$

a. $a=0, c=0$ are asymptotes

b. cost >0

awards >0

Unit 4
Day 8

26. **Open-Ended** Write an equation for a horizontal translation of $y = \frac{2}{x}$. Then write an equation for a vertical translation of $y = \frac{2}{x}$. Identify the horizontal and vertical asymptotes of the graph of each function.

example
26. $y = \frac{2}{x-2}$
h.a: $x=2$
v.a: $y=0$

$y = \frac{2}{x} - 3$
h.a: $x=0$
v.a: $y=-3$

Write each equation in the form $y = \frac{k}{x}$.

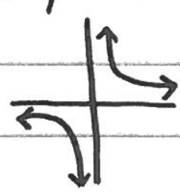
27. $y = \frac{1}{2x}$ **28.** $y = \frac{3}{4x}$ **29.** $y = -\frac{25}{3x}$
30. $xy = -0.01$ **31.** $3xy = 12$ **32.** $-7 = 5xy$

Sketch the graph of each function.

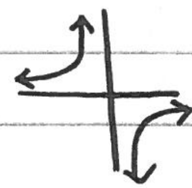
33. $xy = 3$ **34.** $xy + 5 = 0$ **35.** $3xy = 1$
36. $5xy = 2$ **37.** $10xy = -4$ **38.** $3xy = -17$

27. $y = \frac{1/2}{x}$
28. $y = \frac{3/4}{x}$
29. $y = \frac{-25/3}{x}$
30. $y = \frac{-0.01}{x}$
31. $y = \frac{4}{x}$
32. $y = \frac{-7/5}{x}$

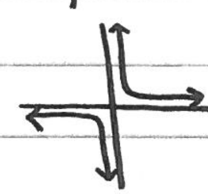
33. $y = 3/x$



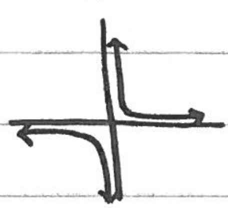
34. $xy = -5$



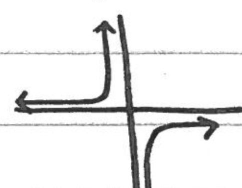
35. $xy = 1/3$



36. $xy = 2/5$



37. $xy = -4/10$



Unit 4
Day 9

Applications

1. The tenth-grade class officers at Columbus High School want to have a special event to welcome the incoming ninth-grade students. For \$1,500, they can rent the Big Ten entertainment center for an evening. Their question is what to charge for tickets to the event so that income from ticket sales will be very close to the rental charge.

   a. Complete a table illustrating the pattern relating number of ticket sales *n* required to meet the "break-even" goal to the price charged *p*. Then write a rule relating *n* to *p*.

Price p (in dollars)	1	3	6	9	12	15
Ticket Sales Needed n	1,500	500				

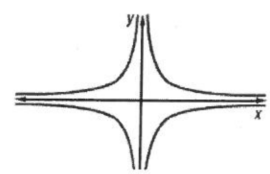
   b. Study entries in the following table showing the class officers' ideas about how price charged p will affect number of students s who will buy tickets to the event. Then write a rule relating s to p.

Price p (in dollars)	0	3	6	9	12	15
Ticket Sales Needed n	600	540	480	420	360	300

   c. Write and solve an equation that will identify the ticket price(s) that will attract enough students for the event to meet its income goal. Illustrate your solution by a sketch of the graphs of the functions involved with key intersection points labeled by their coordinates.

2. When two different students were asked to solve the equation $\frac{3}{x} = -\frac{2}{x}$, they came up with different answers.

Jim argued that there are no values of x that satisfy the equation. He sketched a graph of the two functions to support his claim.



Linda gave the following "proof" that x = 0 is the solution.

If $\frac{3}{x} = \frac{-2}{x}$, then $\frac{x}{3} = \frac{x}{-2}$

then $\frac{x}{3} + \frac{x}{2} = 0$

then $\frac{5x}{6} = 0$

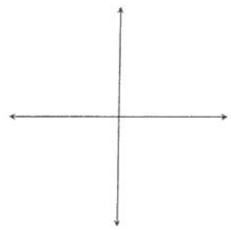
then $x = 0$.

   a. Which student do you think is right – the student who used the graph or the student who used symbolic reasoning?

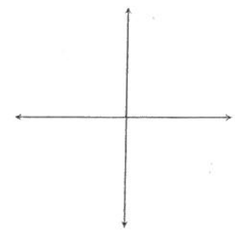
   b. What is the error in reasoning by the student who got the incorrect answer?

3. Use symbolic reasoning to find all solutions for these equations. Illustrate each solution by a sketch of the graphs of the functions involved, labeling key points with their coordinates.

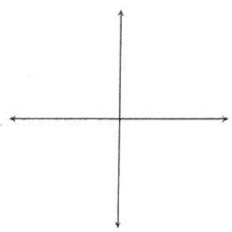
a. $x+5=\dfrac{6}{x}$



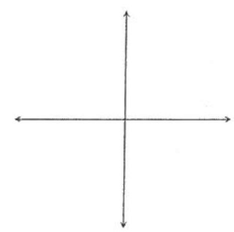
b. $-0.5x=\dfrac{4}{x}$



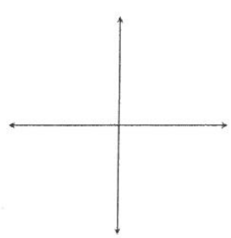
c. $1.5x=\dfrac{24}{x}$



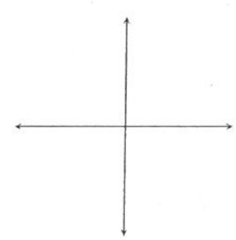
d. $10-x=\dfrac{7}{x}$



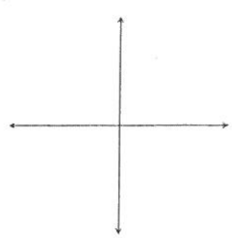
e. $2x=2x^2-4x$



f. $2x^2-4x=4-2x$



g. $x^2-4x-5=2x+2$



h. $-3-x=x^2+3x+1$