

Domain & Range

domain: complete set of possible values
of the independent variable
- "x-values that work" in the function

range: complete set of all possible resulting
values of the dependent variable
- "y-values we get after plugging in
all possible x-values"

* set notation
 $x: 0 \leq x < 3$

* interval notation
 $[0, 3)$

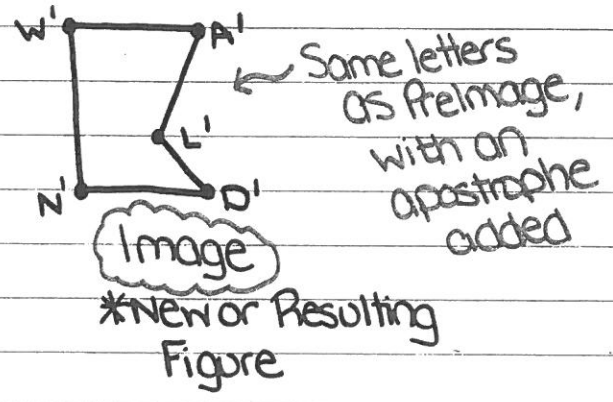
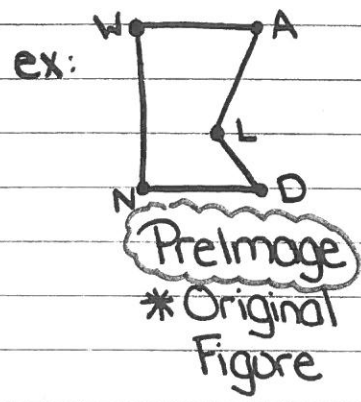
Domain/Range
Activity



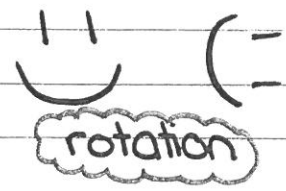
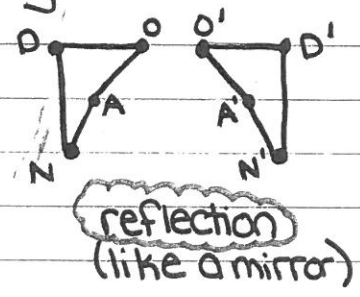
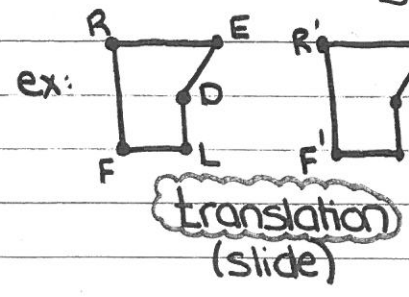
Vocabulary

Congruent Figures: figures that are the same size and shape... when 2 figures are congruent, you can move one so that it fits exactly onto the other.

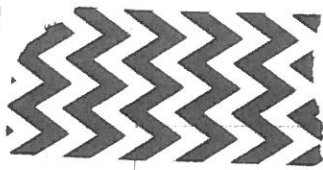
Transformation: change of a geometric figure's position, shape, or size



Isometry: transformation in which the pre-image and image are congruent



Reflections

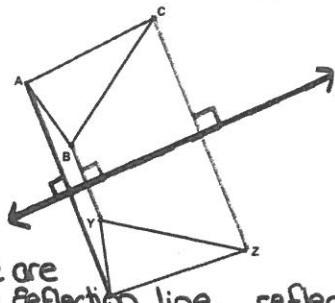


Unit 2
Day 5

Reflections

Reflection Exploration

- 1) $\triangle ABC$ and $\triangle XYZ$ are reflections of each other. While holding the paper towards the light, fold the paper so that the triangles coincide (line up on top of each other). Crease the fold. Then open your paper back up and trace over this fold line using a straightedge to keep it neat.

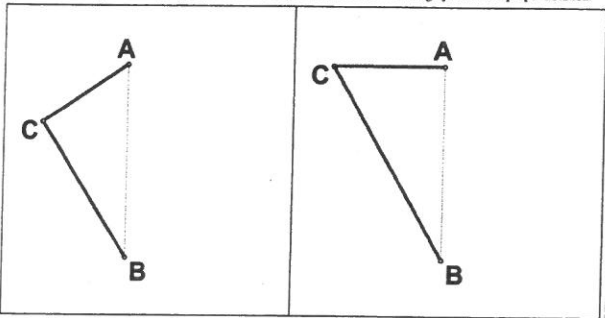


- 2) Using a straightedge, draw \overline{AX} , \overline{BY} , and \overline{CZ} . Look at each segment in relationship to the reflection line. What appears to be true about the reflection line? Discuss lengths of segments and angles created in relationship to the reflection line.

$\overline{AX} \parallel \overline{BY} \parallel \overline{CZ}$ ← these are all \perp to reflection line... reflection line passes through midpt of \overline{AX} , \overline{BY} , and \overline{CZ}

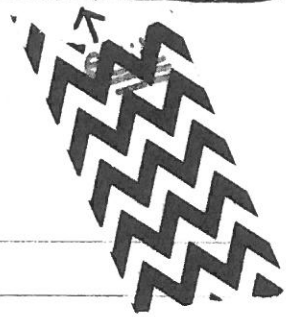
Patty Paper Reflections

Use patty paper to reflect each figure across the dashed line. Transfer the image from the patty paper onto the paper below. Label the image points with proper notation.



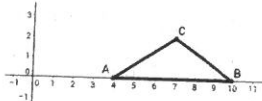
- 3) Points A and B are on the line of reflection. How are A' and B' related to the reflection line?

- 4) Using a straightedge, draw segment $\overline{CC'}$. How is the reflection line related to segment \overline{AB} ?

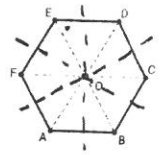


Reflection Symmetry

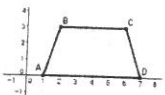
1. Given triangle ABC.
 - a. What is the equation of the line of reflection that maps angle A onto angle B? $x = 7$
 - b. If we reflect triangle ABC over the line of reflection found in part a, AC maps to \overline{BE}
 - c. What can we conclude about the measure of angle A and B? they are =
 - d. What can we conclude about the lengths of \overline{AC} and \overline{BC} ? they are =
 - e. What kind of triangle is ABC? isosceles



2. Given regular hexagon ABCDEF,
 - a. List the three lines of symmetry drawn on the diagram at right: \overline{EF} , \overline{BE} , \overline{AD}
 - b. What is the image of point D when reflected across \overline{BE} ? A
 - c. What is the image of $\angle OED$ when reflected across \overline{FC} ? What conclusions can you make about these angles? $\angle OAB \cong \angle OED$
 - d. Draw the other 3 lines of symmetry not already shown on the diagram.



3. Given quadrilateral ABCD,
 - a. The slope of \overline{BC} is 0. The slope of \overline{AD} is 0. What kind of quadrilateral is ABCD? Explain how you know. trapezoid - only 1 pair of parallel sides



- b. Let line m be the reflection line mapping \overline{CD} to \overline{BA} . Write the equation of line m . $x = 4$
- c. Reflect quadrilateral ABCD over line m .
 - Angle A maps to $\angle D$
 - Angle B maps to $\angle C$

What can be concluded about both pairs of base angles?
 \cong

- d. What is the most specific name of the quadrilateral? Explain how you know. isosceles trapezoid

Activity: Reflections in the coordinate plane. Given $\triangle REF$: $R(-3, 1)$, $E(0, 4)$, $F(2, -5)$

- 1) On the first grid, draw the reflection of $\triangle REF$ in the x -axis. Notation: $R_{x\text{-axis}}$
Record the new coordinates: $R'(\underline{\quad}, \underline{\quad})$, $E'(\underline{\quad}, \underline{\quad})$, $F'(\underline{\quad}, \underline{\quad})$
- 2) On the second grid, draw the reflection of $\triangle REF$ in the y -axis. Notation: $\underline{\hspace{2cm}}$
Record the new coordinates: $R'(\underline{\quad}, \underline{\quad})$, $E'(\underline{\quad}, \underline{\quad})$, $F'(\underline{\quad}, \underline{\quad})$
- 3) Graph the line $y = x$ on the third coordinate grid. Trace $\triangle REF$, both axes, and the line $y = x$ on patty paper. Then flip the patty paper over and line it up again to see where the triangle's image would be if you reflected it in the line $y = x$. Record the new coordinates: $R'(\underline{\quad}, \underline{\quad})$, $E'(\underline{\quad}, \underline{\quad})$, $F'(\underline{\quad}, \underline{\quad})$
- 4) Graph the line $y = -x$ on the fourth coordinate grid. Trace $\triangle REF$, both axes, and the line $y = -x$ on patty paper. Then flip the patty paper over and line it up again to see where the triangle's image would be if you reflected it in the line $y = -x$. Record the new coordinates: $R'(\underline{\quad}, \underline{\quad})$, $E'(\underline{\quad}, \underline{\quad})$, $F'(\underline{\quad}, \underline{\quad})$

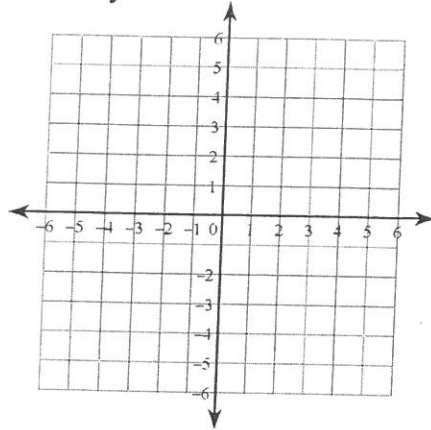
Checkpoint: Look at the patterns and complete the rule. Then write the rule using proper notation.

1. Reflection in the x -axis maps $(x, y) \rightarrow (\underline{\quad}, \underline{\quad})$ Notation: $\underline{\hspace{2cm}}$
2. Reflection in the y -axis maps $(x, y) \rightarrow (\underline{\quad}, \underline{\quad})$ Notation: $\underline{\hspace{2cm}}$
3. Reflection in the line $y = x$ maps $(x, y) \rightarrow (\underline{\quad}, \underline{\quad})$ Notation: $\underline{\hspace{2cm}}$
4. Reflection in the line $y = -x$ maps $(x, y) \rightarrow (\underline{\quad}, \underline{\quad})$ Notation: $\underline{\hspace{2cm}}$

Graph the image using the transformation given, and give the algebraic rule as requested

1. $\triangle EFG$ if $E(-1, 2)$, $F(2, 4)$ and $G(2, -4)$ reflected over the y -axis.

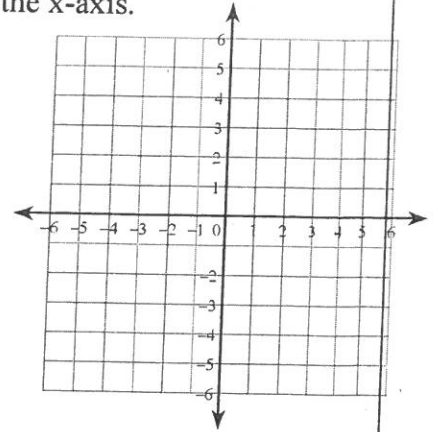
E' _____
 F' _____
 G' _____



Notation:
 Rule:

2. $\triangle PQR$ if $P(-3, 4)$, $Q(4, 4)$ and $R(2, -3)$ reflected over the x -axis.

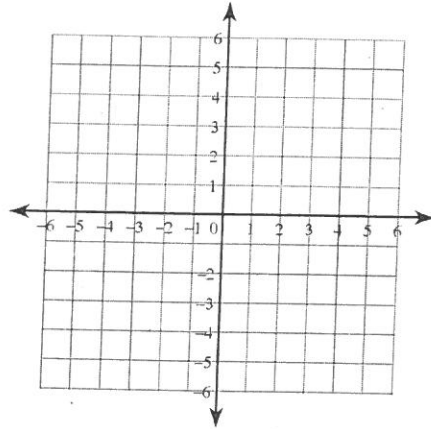
P' _____
 Q' _____
 R' _____



Notation:
 Rule:

3. Quadrilateral $VWXY$ if $V(0, -1)$, $W(1, 1)$, $X(4, -1)$, and $Y(1, -5)$ reflected over the line $y = x$.

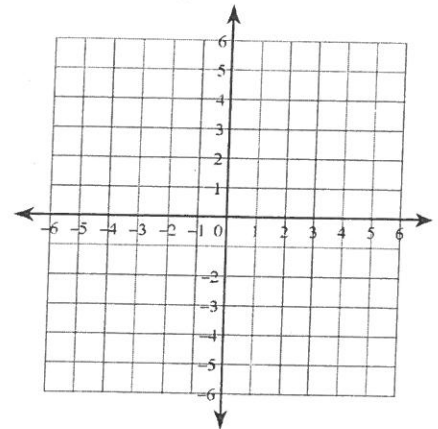
V' _____
 W' _____
 X' _____
 Y' _____



Notation:
 Rule:

4. $\triangle BEL$ if $B(-2, 3)$, $E(2, 4)$, and $L(3, 1)$ reflected over the line $y = -x$.

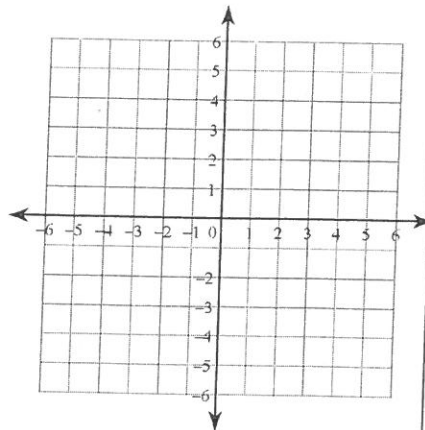
B' _____
 E' _____
 L' _____



Notation:
 Rule:

5. Square $SQUR$ if $S(1, 2)$, $Q(2, 0)$, $U(0, -1)$, and $R(-1, 1)$ reflected over the line $x = 1$.

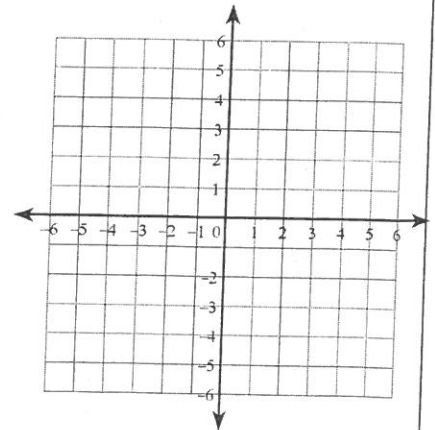
S' _____
 Q' _____
 U' _____
 R' _____



Notation:

6. Quadrilateral $MATH$ if $M(1, 4)$, $A(-1, 2)$, $T(2, 0)$ and $H(4, 0)$ reflected over $y = 2$.

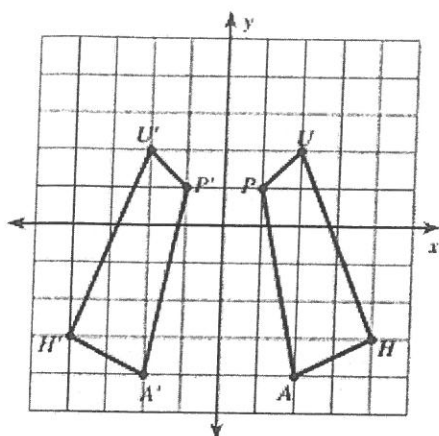
M' _____
 A' _____
 T' _____
 H' _____



Notation:

Write a specific description of each transformation and give the algebraic rule, as requested.

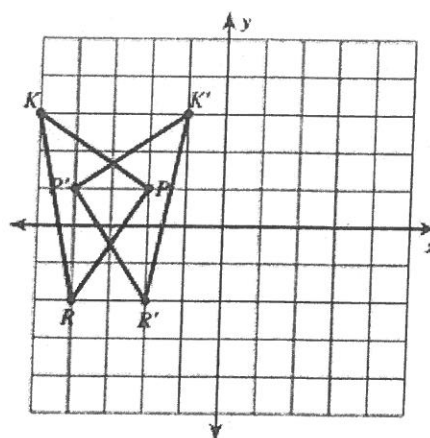
7.



Description:

Algebraic Rule:

8.

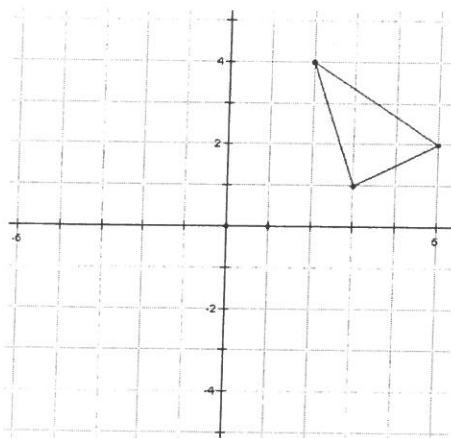


Description:

Find the image of the following transformations and give a specific description.

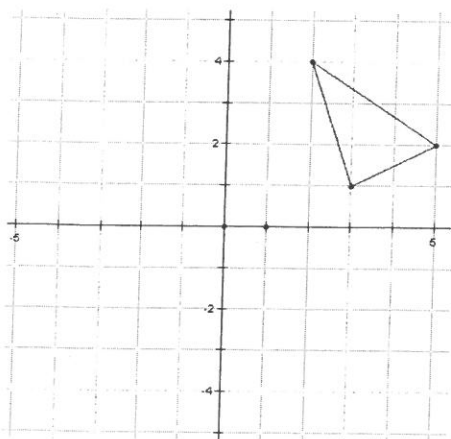
Hint: If you get stuck, review the Checkpoints after today's activities. ☺

9. The points (2,4), (3,1), (5,2) are reflected with the rule $(x, y) \rightarrow (x, -y)$



Description:

10. The points (2,4), (3,1), (5,2) are reflected with the rule $(x, y) \rightarrow (-x, y)$



Description:

Notation:

Translations

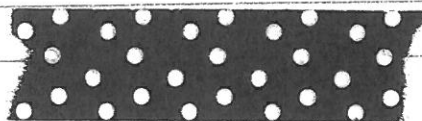
Unit # 2
Day # 6

Three ways to describe a translation:

① words:
move up 2
and right 3

② algebraic rule:
 $(x, y) \rightarrow (x+3, y+2)$

③ vector:
 $\langle 3, 2 \rangle$
↑ describes horizontal movement
↑ describes vertical movement



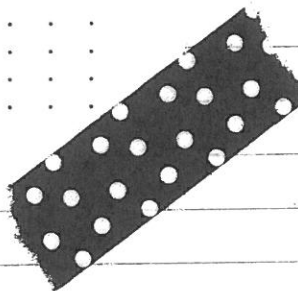
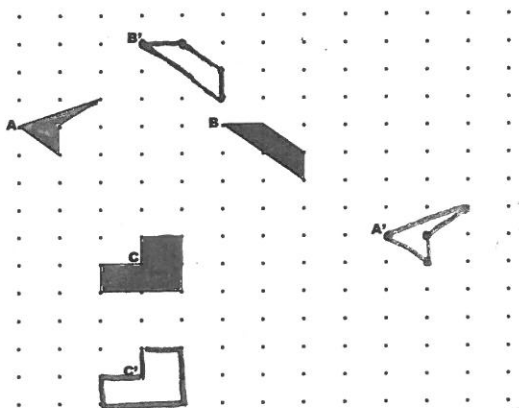
Dot Paper Translations

- 1) Use the dots to help you draw the image of the first figure so that A maps to A'.
- 2) Use the dots to help you draw the image of the second figure so that B maps to B'.
- 3) Use the dots to help you draw the image of the third figure so that C maps to C'.
- 4) Complete each of the following translation rules using your mappings from 1 - 3 above.

a) For A, the translation rule is: $T(x, y) \rightarrow (x+9, y-4)$ or $\langle 9, -4 \rangle$

b) For B, the translation rule is: $T(x, y) \rightarrow (x-2, y+3)$ or $\langle -2, 3 \rangle$

c) For C, the translation rule is: $T(x, y) \rightarrow (x, y+4)$ or $\langle 0, 4 \rangle$



ex: $\triangle GEO$ has coordinates

$$G(-2, 5) \quad E(-4, 1) \quad O(0, -2)$$

A translation maps G to $G'(3, 1)$

(a) Find $E'(\underline{1}, \underline{-3})$ and $O'(\underline{5}, \underline{-6})$

(b) The algebraic rule is $(x, y) \rightarrow \underline{(x+5, y-4)}$

(c) The vector is $\langle \underline{5}, \underline{-4} \rangle$

(d) Use words to specifically describe the translation. move right 5, down 4

ex: $\triangle FRI$ is translated right 2 and down 3, creating image $\triangle F'R'I'$.

Given $FR = 6$	$F'R' = 5x - y$
$RI = 2y$	$R'I' = 4x$
$FI = 10$	$F'I' = 10$

Find: $x = \underline{2}$ $y = \underline{4}$

$6 = 5x - y$	\longleftrightarrow	$6 = 5x - 2x$	$x = 2$
$2y = 4x$	\longleftrightarrow	$y = 2x$	$y = 2 \cdot 2 = 4$
		$6 = 3x$	

*What kind of triangle is $\triangle FRI$? Right \smile $\left. \begin{matrix} FI = 10 \\ FR = 6 \\ RI = 8 \end{matrix} \right\} \text{Pyth. triple!!}$

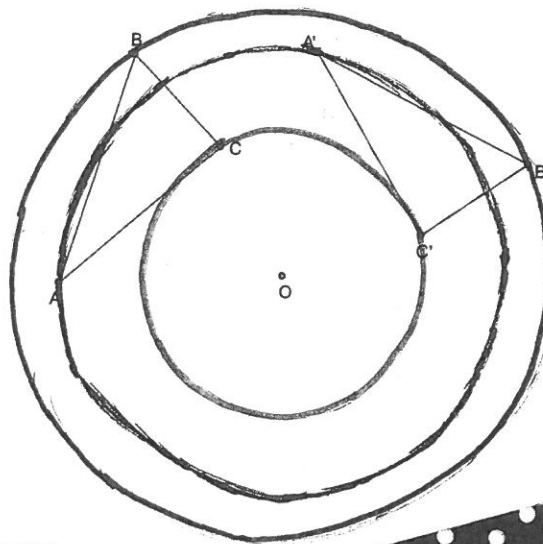
Rotations

Unit 2
Day 7

Rotations Exploration

Triangle $A'B'C'$ is a rotation of Triangle ABC about the center O .

- 1) Using a compass, draw the circle that has center O and goes through point A .
- 2) Using a compass, draw the circle that has center O and goes through point B .
- 3) Using a compass, draw the circle that has center O and goes through point C .
- 4) What do you know notice about points A' , B' , and C' ?
- 5) Trace Triangle ABC and point O on patty paper. Put your pencil point on top of the patty paper on point O and turn the patty paper around and around in both directions (keeping the O on your patty paper on top of the O on this sheet.) What do you notice about the triangle as it rotates around in either direction?



- ④
- A' is on A 's circle
 - B' is on B 's circle
 - C' is on C 's circle

Rotation: a transformation that turns a figure about a given point called the center of rotation

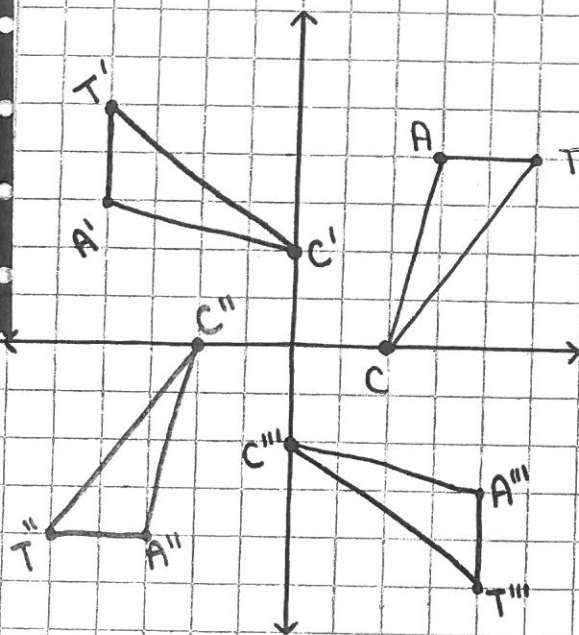
* this point can be inside, on, or outside of the figure

* need to specify: center, angle, direction

* if direction is not specified, then direction is counterclockwise

Rotation Notation: $R_{\text{center, angle}}$

*counterclockwise



C (2,0) A(3,4) T(5,4)

(a) 90° rotation or $R_{0,90^\circ}$

C' (0,2) A' (-4,3) T' (-4,5)

(b) 180° rotation or $R_{0,180^\circ}$

C'' (-2,0) A'' (-3,-4) T'' (-5,-4)

(c) 270° rotation or $R_{0,270^\circ}$

C''' (0,-2) A''' (4,-3) T''' (4,-5)

Rules for rotation about the origin:

90° counterclockwise $(x,y) \rightarrow (-y,x)$

180° counterclockwise $(x,y) \rightarrow (-x,-y)$

270° counterclockwise $(x,y) \rightarrow (y,-x)$

270° ccw \approx 90° cw

90° ccw \approx 270° cw

Rotations with Polygons

Part 1 - Regular Polygons and rotation symmetry

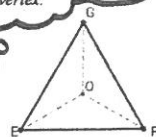
A few definitions to support you as you work:

A regular polygon is a polygon that is equiangular (all angles are equal in measure) and equilateral (all sides have the same length). In the case of regular polygons the center is the point that is equidistant from each vertex.

Great vocabulary to take note of!

1. Given regular triangle EFG with center O.

- a. F is rotated about O. If the image of F is G, what is the angle of rotation? 120°
- b. \overline{FG} is rotated 120° about O. What is the image of \overline{FG} ? \overline{GE}

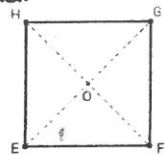


General Rule: The regular triangle has rotation symmetry with respect to the center of the polygon and angles of rotation that measure 120° and 240°

Side note: A regular triangle is also called an equilateral triangle or an equiangular triangle.

2. Given regular quadrilateral EFGH with center O.

- a. F is rotated about O. If the image of F is G, what is the angle of rotation? 90°
- b. F is rotated about O. If the image of F is H, what is the angle of rotation? 180°
- c. \overline{FG} is rotated 270° about O. What is the image of \overline{FG} ? \overline{EH}

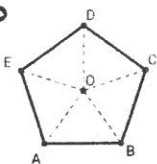


General Rule: The regular quadrilateral has rotation symmetry with respect to the center of the polygon and angles of rotation that measure 90° , 180° , 270° and 360°

Side note: A regular quadrilateral is often called a square

3. Given regular pentagon ABCDE with center O,

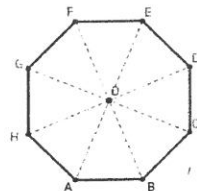
- a. C is rotated about O. If the image of C is D, what is the angle of rotation? 72°
- b. C is rotated about O. If the image of C is E, what is the angle of rotation? 144°
- c. C is rotated about O. If the image of C is A, what is the angle of rotation? 216°
- d. \overline{DC} is rotated 288° about O, what is the image of \overline{DC} ? \overline{BC}
- e. Pentagon ABCDE is rotated 72° about O, what is the image of pentagon ABCDE (in terms of the original points' labels - do not use A'B'C'D'E')? $BCDEA$
- f. Explain the significance of the multiples of 72° . $360 \div 5$



General Rule: The regular pentagon has rotation symmetry with respect to the center of the polygon and angles of rotation that measure 72° , 144° , 216° , 288° and 360°

5. Given regular octagon ABCDEFGH with center O,

- a. When point C is rotated about O, the image of point C is point D. Describe the rotation (be sure to include degree). 45° CCW
- b. When point C is rotated about O, the image of point C is point F. Describe the rotation (be sure to include degree). 135° CCW



A regular polygon can be mapped onto itself if we rotate in multiples of the central angle measure. The central angle of a regular polygon is found by $360 \div \# \text{ of sides}$

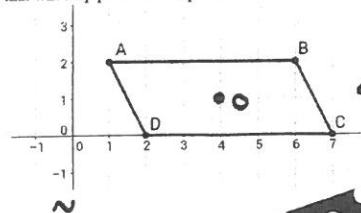
Lift flap... there's a hexagon hiding.

This is a great thing to know!

Part 2 - Parallelograms and rotation symmetry

6. Given parallelogram ABCD, there is a center of rotation, O, that will map point A onto point C.

- a. What are the coordinates of O? $(4, 1)$
- b. What degree of rotation mapped C onto A using the center O? 180°
- c. If we rotate the parallelogram around center O using the degree measure found in part b, angle D maps to angle B .
- d. If angle A maps to angle C, then angle A and angle C are \cong .
- e. If angle D maps to angle B , then angle D and angle B are \cong .



So, opposite angles in a parallelogram are \cong ... hmmm...

