a number that can be expressed as the quotient of 2 integers (p/q)

3.1 Rational Exponents

Unit 3
Day 1

In your calculator, graph:

① $y = \sqrt{x}$
$y = x^{1/2}$

② $y = \sqrt[3]{x}$
$y = x^{1/3}$

③ $y = \sqrt[3]{x^2}$
$y = x^{2/3}$

hmm.... $y = \sqrt[n]{x^m}$ } these are the same.
$y = x^{m/n}$ }

Review:

$x^a \cdot x^b = x^{a+b}$ ex: $x^2 \cdot x^{10} = x^{12}$

$(x^a)^b = x^{a \cdot b}$ ex: $(x^2)^{10} = x^{20}$

$\frac{x^a}{x^b} = x^{a-b}$ ex: $\frac{x^{10}}{x^2} = x^8$

$x^{-a} = \frac{1}{x^a}$ ex: $x^{-2} = \frac{1}{x^2}$

also:
$*x^0 = 1$

Review: $\sqrt{16} = 4$ vs. $x^2 = 16$

$x = \pm 4$

Review: $\sqrt[3]{125} = 5$ because $5 \cdot 5 \cdot 5 = 5^3 = 125$

$\sqrt[4]{16} = 2$ because $2 \cdot 2 \cdot 2 \cdot 2 = 2^4 = 16$

$\sqrt[3]{-8} = -2$ because $-2 \cdot -2 \cdot -2 = -8$

New: $x^{m/n} = \sqrt[n]{x^m}$

ex: $8^{5/3} = \sqrt[3]{8^5}$
$= (\sqrt[3]{8})^5$
$= 2^5 = 32$

Unit 3
Day 1

Put it all together

Simplify: $(-32)^{3/5} = \left[(-32)^{1/5}\right]^3$

$= (-2)^3$

$= \boxed{-8}$

or... $(-32)^{3/5} = \sqrt[5]{(-32)^3} = \left(\sqrt[5]{-32}\right)^3$

$= (-2)^3$

$= \boxed{-8}$

Simplify: $4^{-3.5} = \frac{1}{4^{3.5}} = \frac{1}{4^{7/2}} = \frac{1}{(4^{1/2})^7}$

$= \frac{1}{2^7}$

$= \boxed{\frac{1}{128}}$

or... $4^{-3.5}$

$= \frac{1}{4^{3.5}} = \frac{1}{4^{7/2}} = \frac{1}{\sqrt{4^7}} = \boxed{\frac{1}{128}}$

Simplify: $\left(16y^{-8}\right)^{-3/4} = 16^{-3/4} \cdot \left(y^{-8}\right)^{-3/4}$

$= \frac{1}{16^{3/4}} \cdot y^6$

$= \left(\frac{1}{\sqrt[4]{16}}\right)^3 \cdot y^6$

$= \frac{1}{2^3} \cdot y^6 = \boxed{\frac{y^6}{8}}$

Unit 3
Day 1

Your turn: Simplify

1. $36^{1/2} = 6$

2. $27^{1/3} = 3$

3. $3^{1/2} \cdot 12^{1/2} = 6$

4. $3^{1/3} \cdot 9^{1/3} = 3$

5. $3^{1/4} \cdot 27^{1/4} = 3$

6. $8^{2/3} = 4$

7. $(-8)^{2/3} = 4$

8. $10000^{3/4} = 1000$

9. $16^{1.5} = 64$

10. $(32)^{-4/5} = \frac{1}{16}$

Write each in Radical Form:

11. $x^{1/6} = \sqrt[6]{x}$

12. $t^{-3/4} = \frac{1}{\sqrt[4]{t^3}}$

13. $y^{1.2} = \sqrt[5]{y^6}$

14. $y^{9/8} = \sqrt[8]{y^9}$

Write each in exponential form:

15. $\sqrt{-10} = (-10)^{1/2}$

16. $\sqrt[3]{a^2} = a^{2/3}$

17. $\sqrt{7x^3} = 7^{1/2} x^{3/2}$

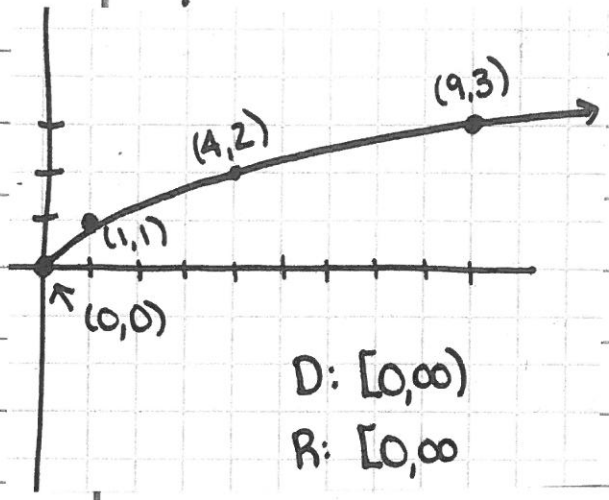
18. $\sqrt[3]{(5xy)^6} = (5xy)^2$

3.2 Graphing Radical Functions

Unit 3
Day 2

"Parent Functions" we need to know:

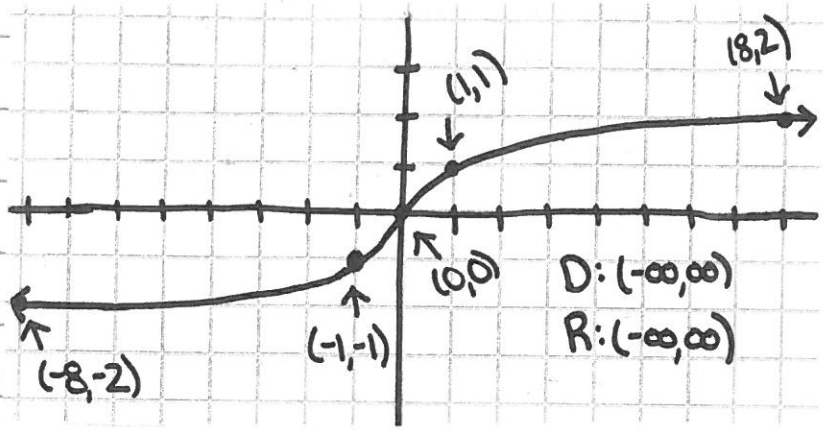
$y = \sqrt{x}$



D: $[0,\infty)$

R: $[0,\infty)$

$y = \sqrt[3]{x}$



D: $(-\infty,\infty)$

R: $(-\infty,\infty)$

Review: $y = f(x)$ is our parent graph

$y = f(x+2)$	$f(x)$ shifts left 2
$y = f(x-2)$	$f(x)$ shifts right 2
$y = f(x)+2$	$f(x)$ shifts up 2
$y = f(x)-2$	$f(x)$ shifts down 2
$y = -f(x)$	$f(x)$ is reflected across x-axis
$y = f(-x)$	$f(x)$ is reflected across y-axis
$y = 2f(x)$	$f(x)$ is vertically stretched
$y = \frac{1}{2}f(x)$	$f(x)$ is vertically reduced

Unit 3
Day 2

Without a calculator, graph the following. Be sure to plot at least 3 points. *State the domain and range.

1. $y=\sqrt{x-1}-5$

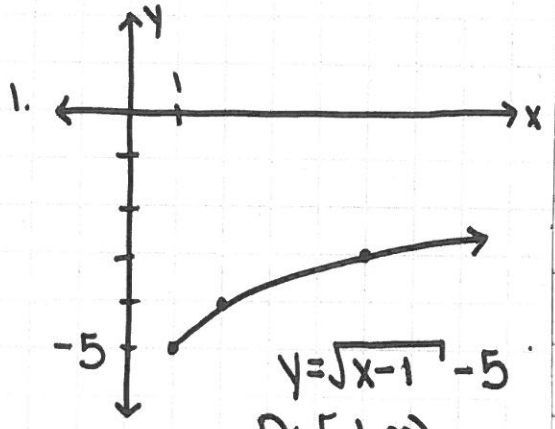
2. $y=-\sqrt{x-2}+5$

3. $y=\sqrt[3]{x+1}$

4. $y=\sqrt[3]{x+1}+2$

Fun ☺ $y=\sqrt{4x-12}$

1.

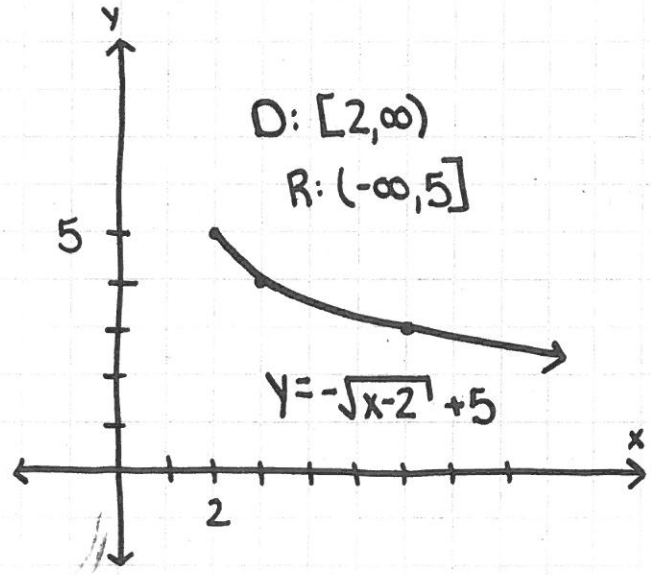


$y=\sqrt{x-1}-5$

D: $[1,\infty)$

R: $[-5,\infty)$

2.

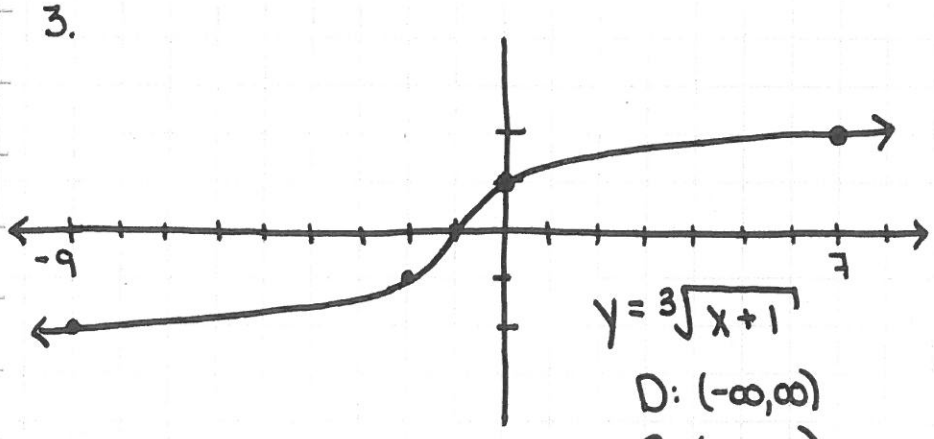


D: $[2,\infty)$

R: $(-\infty,5]$

$y=-\sqrt{x-2}+5$

3.

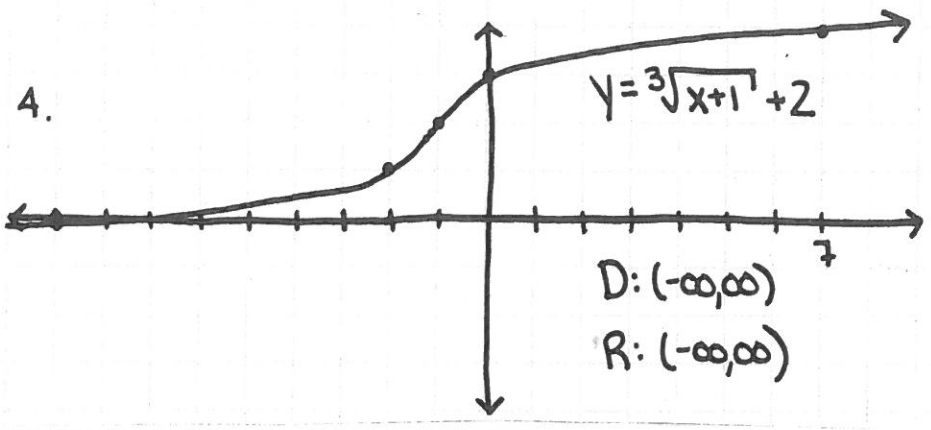


$y=\sqrt[3]{x+1}$

D: $(-\infty,\infty)$

R: $(-\infty,\infty)$

4.

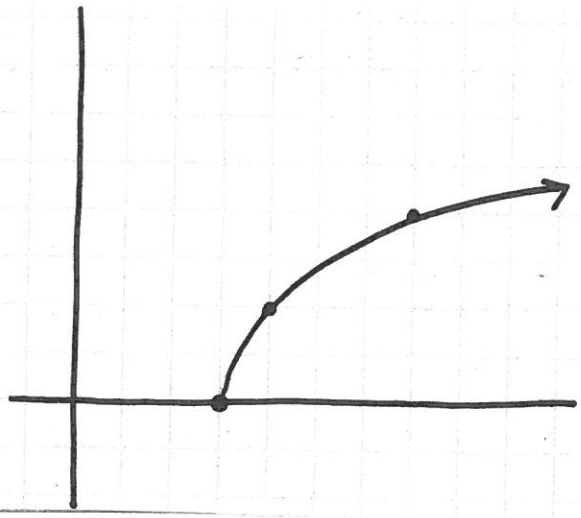


5. $y=\sqrt{4x-12}$

$y=\sqrt{4(x-3)}$

$y=2\sqrt{x-3}$

x	y
3	0
4	2
7	4



3.3 Solving Radical Equations

Unit 3
Day 3

Radical Equation - Equation that has a variable under the radical or that has a variable with a rational exponent.

ex. Solve $\sqrt{2x-1}-3=0$

First, Isolate the radical

$\sqrt{2x-1}=3$

Then, Square each side

$2x-1=9$

$2x=10$

$\boxed{x=5}$

*Check your answer!!

$\sqrt{2\cdot5-1}-3\stackrel{?}{=}0$

$\sqrt{9}-3\stackrel{?}{=}0$ ✓!!

ex. Solve $2(2x)^{1/3}+1=5$

First, Isolate the radical

$2(2x)^{1/3}=4$

$(2x)^{1/3}=2$

Then, Cube both sides

$\sqrt[3]{2x}=2$

$2x=8$

$\boxed{x=4}$

*Check your answer!!

$2(2\cdot4)^{1/3}+1\stackrel{?}{=}5$

$2\cdot8^{1/3}+1\stackrel{?}{=}5$

$2\cdot2+1=5$ ✓!!

Unit 3
Day 3

ex: $\sqrt{11x+3} - 2x = 0$

$\sqrt{11x+3} = 2x$

$11x+3 = 4x^2$

$0 = 4x^2 - 11x - 3$

$0 = (4x+1)(x-3)$

$x = -\frac{1}{4}$ $\boxed{x=3}$

check:

$x=-\frac{1}{4}$: $\sqrt{11\cdot-\frac{1}{4}+3} - 2\cdot-\frac{1}{4} \stackrel{?}{=} 0$

$\sqrt{-\frac{11}{4}+\frac{12}{4}} + \frac{1}{2} \stackrel{?}{=} 0$

$\sqrt{\frac{1}{4}} + \frac{1}{2} \stackrel{?}{=} 0$

No

$x=3$: $\sqrt{33+3} - 2\cdot3 \stackrel{?}{=} 0$

$0 = 0$ ✓

you try:

① $3\sqrt{x} + 3 = 15$ — $x = 16$

② $4\sqrt{x} - 1 = 3$ — $x = 1$

③ $\sqrt{x+3} = 5$ — $x = 22$

④ $(x+5)^{1/2} = 4$ — $x = 11$

⑤ challenger ☺

$(x+5)^{2/3} = 4$ — $x = 3$

~~⑥ $(5x+4)^{1/2} - 3x = 0$~~

⑥ $(5x+4)^{1/2} - 3x = 0$ — $x = 1$

~~$x = -\frac{4}{9}$~~

Unit 3
Day 3

Challenge:

1. $\sqrt{x^2+3} = x+1$

2. $\sqrt{4x-10} = 3\sqrt{x-5}$

3. $\sqrt{x} = x-2$

4. $x+8 = 4\sqrt{x+5}$

1. $x^2+3 = x^2+2x+1$

$3 = 2x+1$

$2 = 2x$

$\boxed{1 = x}$

2. $4x-10 = 9x-45$

$35 = 5x$

$\boxed{7 = x}$

3. $x = x^2-4x+4$

$0 = x^2-5x+4$

$0 = (x-4)(x-1)$

$\boxed{x=4}$, ~~$x=1$~~

4. $x^2+16x+64 = 16x+80$

~~$x^2+16x+64 = 16x+80$~~

~~$x^2-16=0$~~

~~$x=4,-4$~~

$\boxed{x=-4}$