Extension
Use after Lesson 7.7

Law of Sines and Law of Cosines

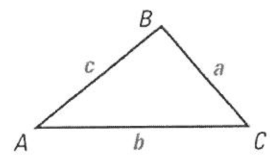
GOAL Use trigonometry with acute and obtuse triangles.

The trigonometric ratios you have seen so far in this chapter can be used to find angle and side measures in right triangles. You can use the Law of Sines to find angle and side measures in *any* triangle.

KEY CONCEPT — *For Your Notebook*

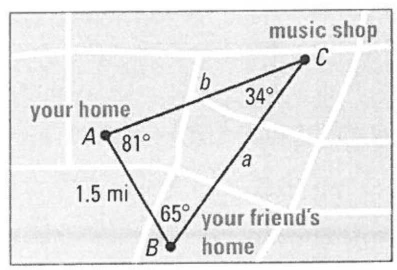
Law of Sines

If $\triangle ABC$ has sides of length a, b, and c as shown, then $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.



EXAMPLE 1 Find a distance using Law of Sines

DISTANCE Use the information in the diagram to determine how much closer you live to the music store than your friend does.



Solution

STEP 1 **Use** the Law of Sines to find the distance a from your friend's home to the music store.

$\frac{\sin A}{a} = \frac{\sin C}{c}$ **Write Law of Sines.**

$\frac{\sin 81°}{a} = \frac{\sin 34°}{1.5}$ **Substitute.**

$a \approx 2.6$ **Solve for *a*.**

STEP 2 **Use** the Law of Sines to find the distance b from your home to the music store.

$\frac{\sin B}{b} = \frac{\sin C}{c}$ **Write Law of Sines.**

$\frac{\sin 65°}{b} = \frac{\sin 34°}{1.5}$ **Substitute.**

$b \approx 2.4$ **Solve for *b*.**

STEP 3 **Subtract** the distances.

$a - b \approx 2.6 - 2.4 = 0.2$

▶ You live about 0.2 miles closer to the music store.

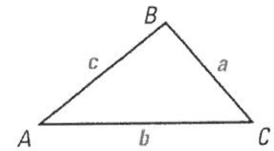
LAW OF COSINES You can also use the Law of Cosines to solve any triangle.

KEY CONCEPT *For Your Notebook*

Law of Cosines

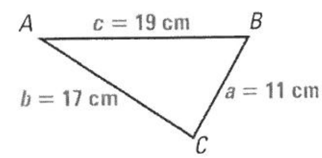
If $\triangle ABC$ has sides of length a, b, and c, then:

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$



EXAMPLE 2 Find an angle measure using Law of Cosines

In $\triangle ABC$ at the right, $a = 11$ cm, $b = 17$ cm, and $c = 19$ cm. Find $m\angle C$.



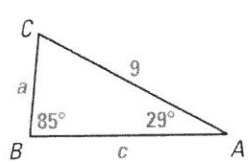
Solution

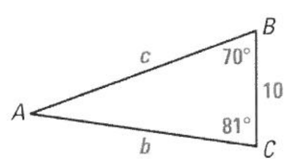
$c^2 = a^2 + b^2 - 2ab \cos C$	Write Law of Cosines.
$19^2 = 11^2 + 17^2 - 2(11)(17) \cos C$	Substitute.
$0.1310 = \cos C$	Solve for cos *C*.
$m\angle C \approx 82°$	Find $\cos^{-1}(0.1310)$.

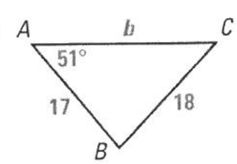
PRACTICE

EXAMPLE 1 for Exs. 1–3

LAW OF SINES Use the Law of Sines to solve the triangle. Round decimal answers to the nearest tenth.

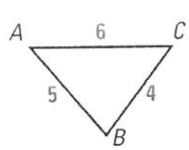
1. 

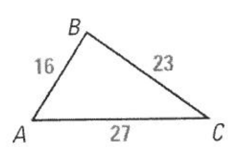
2. 

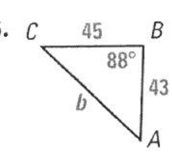
3. 

EXAMPLE 2 for Exs. 4–7

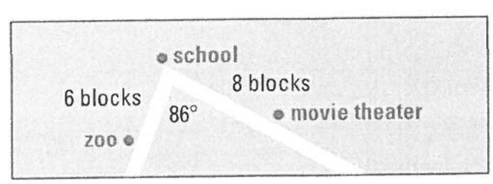
LAW OF COSINES Use the Law of Cosines to solve the triangle. Round decimal answers to the nearest tenth.

4. 

5. 

6. 

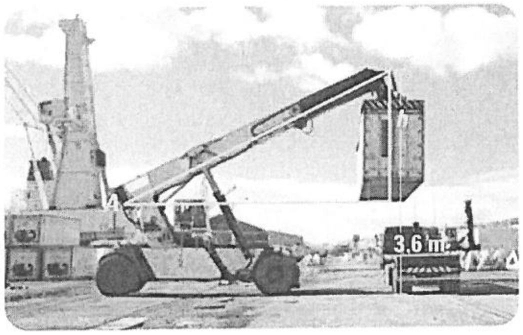
7. **DISTANCE** Use the diagram at the right. Find the straight distance between the zoo and movie theater.



MIXED REVIEW of Problem Solving

Lessons 7.5–7.7

1. **MULTI-STEP PROBLEM** A *reach stacker* is a vehicle used to lift objects and move them between ships and land.



   a. The vehicle's arm is 10.9 meters long. The maximum measure of $\angle A$ is 60°. What is the greatest height h the arm can reach if the vehicle is 3.6 meters tall?

   b. The vehicle's arm can extend to be 16.4 meters long. What is the greatest height its extended arm can reach?

   c. What is the difference between the two heights the arm can reach above the ground?

2. **EXTENDED RESPONSE** You and a friend are standing the same distance from the edge of a canyon. Your friend looks directly across the canyon at a rock. You stand 10 meters from your friend and estimate the angle between your friend and the rock to be 85°.

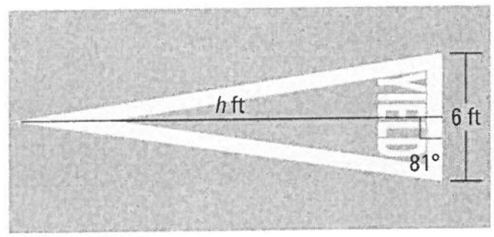
   a. Sketch the situation.

   b. *Explain* how to find the distance across the canyon.

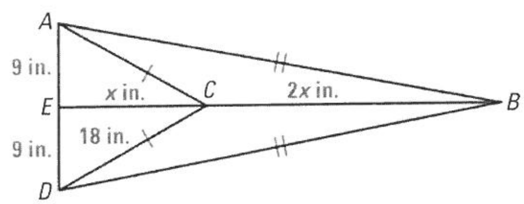
   c. Suppose the actual angle measure is 87°. How far off is your estimate of the distance?

3. **SHORT RESPONSE** The international rules of basketball state the rim of the net should be 3.05 meters above the ground. If your line of sight to the rim is 34° and you are 1.7 meters tall, what is the distance from you to the rim? *Explain* your reasoning.

4. **GRIDDED ANSWER** The specifications for a *yield ahead* pavement marking are shown. Find the height h in feet of this isosceles triangle.



5. **EXTENDED RESPONSE** Use the diagram to answer the questions.

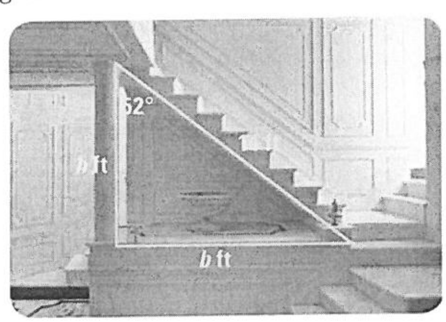


   a. Solve for x. *Explain* the method you chose.

   b. Find $m\angle ABC$. *Explain* the method you chose.

   c. *Explain* a different method for finding each of your answers in parts (a) and (b).

6. **SHORT RESPONSE** The triangle on the staircase below has a 52° angle and the distance along the stairs is 14 feet. What is the height h of the staircase? What is the length b of the base of the staircase?



7. **GRIDDED ANSWER** The base of an isosceles triangle is 70 centimeters long. The altitude to the base is 75 centimeters long. Find the measure of a base angle to the nearest degree.

7 CHAPTER SUMMARY

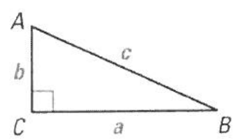
BIG IDEAS — For Your Notebook

Big Idea 1

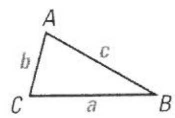
Using the Pythagorean Theorem and Its Converse

The Pythagorean Theorem states that in a right triangle the square of the length of the hypotenuse c is equal to the sum of the squares of the lengths of the legs a and b, so that $c^2 = a^2 + b^2$.

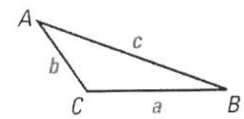
The Converse of the Pythagorean Theorem can be used to determine if a triangle is a right triangle.



If $c^2 = a^2 + b^2$, then $m\angle C = 90°$ and $\triangle ABC$ is a right triangle.



If $c^2 < a^2 + b^2$, then $m\angle C < 90°$ and $\triangle ABC$ is an acute triangle.



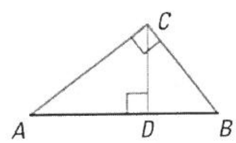
If $c^2 > a^2 + b^2$, then $m\angle C > 90°$ and $\triangle ABC$ is an obtuse triangle.

Big Idea 2

Using Special Relationships in Right Triangles

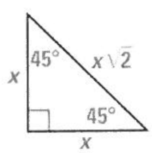
GEOMETRIC MEAN In right $\triangle ABC$, altitude $\overline{CD}$ forms two smaller triangles so that $\triangle CBD \sim \triangle ACD \sim \triangle ABC$.

Also, $\frac{BD}{CD} = \frac{CD}{AD}$, $\frac{AB}{CB} = \frac{CB}{DB}$, and $\frac{AB}{AC} = \frac{AC}{AD}$.



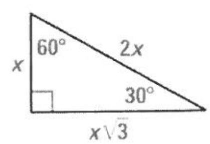
SPECIAL RIGHT TRIANGLES

45°-45°-90° Triangle



hypotenuse = leg · $\sqrt{2}$

30°-60°-90° Triangle



hypotenuse = 2 · shorter leg
longer leg = shorter leg · $\sqrt{3}$

Big Idea 3

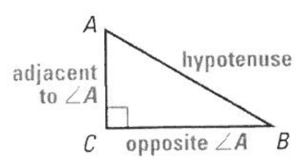
Using Trigonometric Ratios to Solve Right Triangles

The tangent, sine, and cosine ratios can be used to find unknown side lengths and angle measures of right triangles. The values of tan $x°$, sin $x°$, and cos $x°$ depend only on the angle measure and not on the side length.

$\tan A = \frac{\text{opp.}}{\text{adj.}} = \frac{BC}{AC}$ $\qquad \tan^{-1} \frac{BC}{AC} = m\angle A$

$\sin A = \frac{\text{opp.}}{\text{hyp.}} = \frac{BC}{AB}$ $\qquad \sin^{-1} \frac{BC}{AB} = m\angle A$

$\cos A = \frac{\text{adj.}}{\text{hyp.}} = \frac{AC}{AB}$ $\qquad \cos^{-1} \frac{AC}{AB} = m\angle A$



7 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

- Pythagorean triple, *p. 435*
- trigonometric ratio, *p. 466*
- tangent, *p. 466*
- sine, *p. 473*
- cosine, *p. 473*
- angle of elevation, *p. 475*
- angle of depression, *p. 475*
- solve a right triangle, *p. 483*
- inverse tangent, *p. 483*
- inverse sine, *p. 483*
- inverse cosine, *p. 483*

VOCABULARY EXERCISES

1. Copy and complete: A Pythagorean triple is a set of three positive integers a, b, and c that satisfy the equation __?__.
2. **WRITING** What does it mean to solve a right triangle? What do you need to know to solve a right triangle?
3. **WRITING** *Describe* the difference between an angle of depression and an angle of elevation.

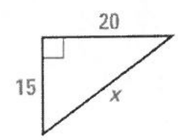
REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 7.

7.1 Apply the Pythagorean Theorem *pp. 433–439*

EXAMPLE

Find the value of x.



Because x is the length of the hypotenuse of a right triangle, you can use the Pythagorean Theorem to find its value.

$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$ **Pythagorean Theorem**

$x^2 = 15^2 + 20^2$ **Substitute.**

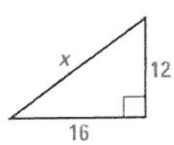
$x^2 = 625$ **Simplify.**

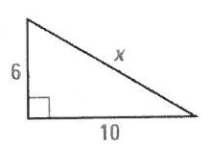
$x = 25$ **Find the positive square root.**

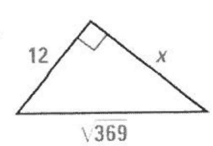
EXERCISES

EXAMPLES 1 and 2 on pp. 433–434 for Exs. 4–6

Find the unknown side length x.

4. 

5. 

6. 

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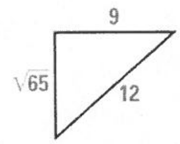
Chapter Review Practice

7.2 Use the Converse of the Pythagorean Theorem *pp. 441–447*

EXAMPLE

Tell whether the given triangle is a right triangle.



Check to see whether the side lengths satisfy the equation $c^2 = a^2 + b^2$.

$12^2 \stackrel{?}{=} (\sqrt{65})^2 + 9^2$

$144 \stackrel{?}{=} 65 + 81$

$144 < 146$

The triangle is not a right triangle. It is an acute triangle.

EXERCISES

EXAMPLE 2 on p. 442 for Exs. 7–12

Classify the triangle formed by the side lengths as *acute, right,* or *obtuse.*

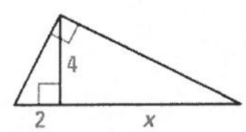
7. 6, 8, 9 **8.** 4, 2, 5 **9.** 10, $2\sqrt{2}$, $6\sqrt{3}$

10. 15, 20, 15 **11.** 3, 3, $3\sqrt{2}$ **12.** 13, 18, $3\sqrt{55}$

7.3 Use Similar Right Triangles *pp. 449–456*

EXAMPLE

Find the value of *x*.



By Theorem 7.6, you know that 4 is the geometric mean of x and 2.

$\frac{x}{4} = \frac{4}{2}$ **Write a proportion.**

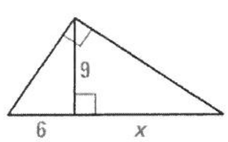
$2x = 16$ **Cross Products Property**

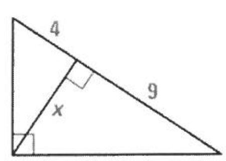
$x = 8$ **Divide.**

EXERCISES

EXAMPLES 2 and 3 on pp. 450–451 for Exs. 13–18

Find the value of *x*.

13.


14.


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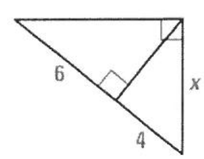

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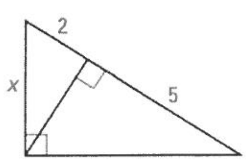
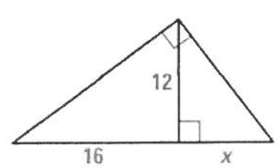
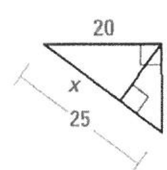


17.


18.


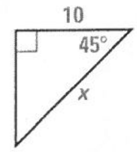
7 CHAPTER REVIEW

7.4 Special Right Triangles

pp. 457–464

EXAMPLE

Find the length of the hypotenuse.



By the Triangle Sum Theorem, the measure of the third angle must be 45°. Then the triangle is a 45°-45°-90° triangle.

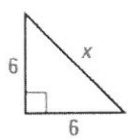
hypotenuse $=$ leg $\cdot \sqrt{2}$		**45°-45°-90° Triangle Theorem**
$x = 10\sqrt{2}$		**Substitute.**

EXERCISES

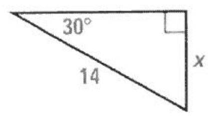
EXAMPLES 1, 2, and 5 on pp. 457–459 for Exs. 19–21

Find the value of x. Write your answer in simplest radical form.

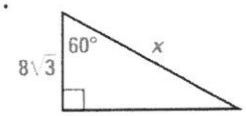
19.



20.



21.

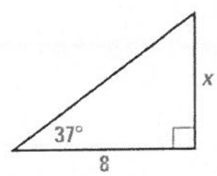


7.5 Apply the Tangent Ratio

pp. 466–472

EXAMPLE

Find the value of x.

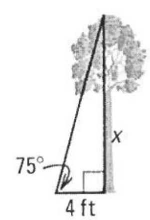


$\tan 37° = \frac{\text{opp.}}{\text{adj.}}$	**Write ratio for tangent of 37°.**
$\tan 37° = \frac{x}{8}$	**Substitute.**
$8 \cdot \tan 37° = x$	**Multiply each side by 8.**
$6 \approx x$	**Use a calculator to simplify.**

EXERCISES

EXAMPLE 2 on p. 467 for Exs. 22–26

In Exercises 22 and 23, use the diagram.

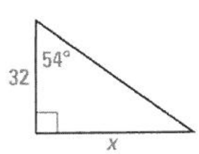


22. The angle between the bottom of a fence and the top of a tree is 75°. The tree is 4 feet from the fence. How tall is the tree? Round your answer to the nearest foot.

23. In Exercise 22, how tall is the tree if the angle is 55°?

Find the value of x to the nearest tenth.

24.



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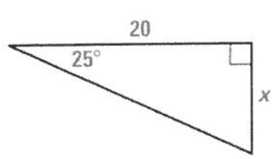
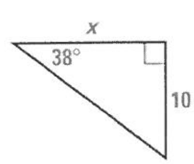


26.



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Chapter Review Practice

7.6 Apply the Sine and Cosine Ratios

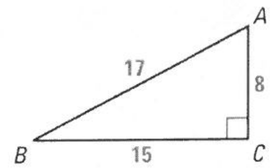
pp. 473–480

EXAMPLE

Find sin A and sin B.

$\sin A = \frac{\text{opp.}}{\text{hyp.}} = \frac{BC}{BA} = \frac{15}{17} \approx 0.8824$

$\sin B = \frac{\text{opp.}}{\text{hyp.}} = \frac{AC}{AB} = \frac{8}{17} \approx 0.4706$

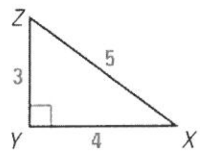


EXERCISES

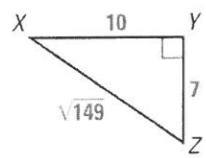
EXAMPLES 1 and 2 on pp. 473–474 for Exs. 27–29

Find sin X and cos X. Write each answer as a fraction, and as a decimal. Round to four decimals places, if necessary.

27.



28.



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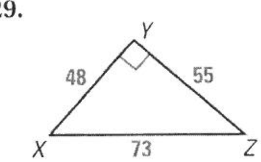
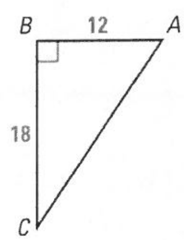


7.7 Solve Right Triangles

pp. 483–489

EXAMPLE

Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.



Because $\tan A = \frac{18}{12} = \frac{3}{2} = 1.5$, $\tan^{-1} 1.5 = m\angle A$.

Use a calculator to evaluate this expression.

$\tan^{-1} 1.5 \approx 56.3099324\ldots$

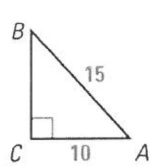
So, the measure of $\angle A$ is approximately 56.3°.

EXERCISES

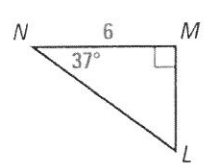
EXAMPLE 3 on p. 484 for Exs. 30–33

Solve the right triangle. Round decimal answers to the nearest tenth.

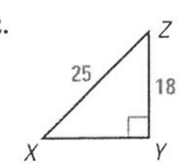
30.



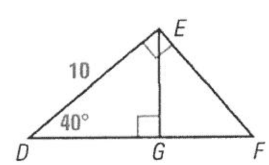
31.



32.

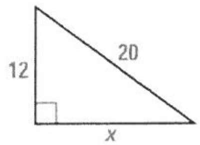


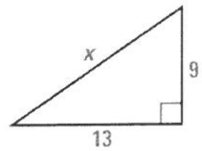
33. Find the measures of $\angle GED$, $\angle GEF$, and $\angle EFG$. Find the lengths of $\overline{EG}$, $\overline{DF}$, $\overline{EF}$.

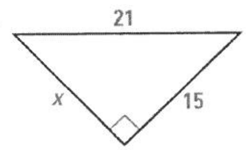


7 CHAPTER TEST

Find the value of x. Write your answer in simplest radical form.

1. 

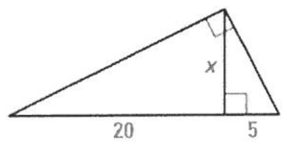
2. 

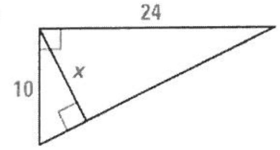
3. 

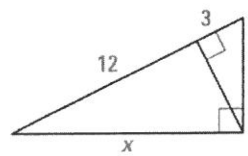
Classify the triangle as *acute, right,* or *obtuse.*

4. $5, 15, 5\sqrt{10}$
5. 4.3, 6.7, 8.2
6. 5, 7, 8

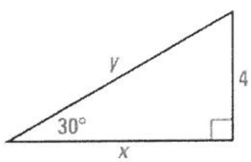
Find the value of x. Round decimal answers to the nearest tenth.

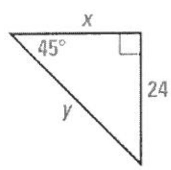
7. 

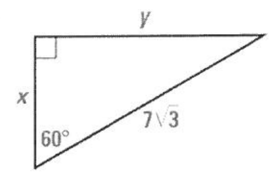
8. 

9. 

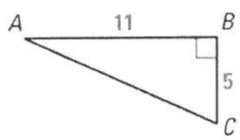
Find the value of each variable. Write your answer in simplest radical form.

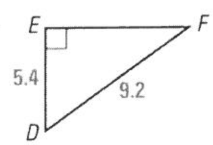
10. 

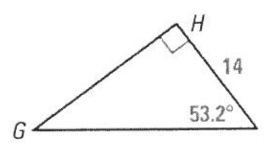
11. 

12. 

Solve the right triangle. Round decimal answers to the nearest tenth.

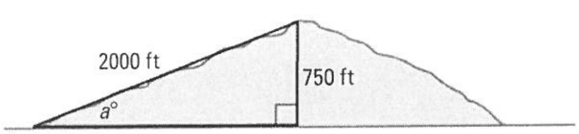
13. 

14. 

15. 

16. **FLAGPOLE** Julie is 6 feet tall. If she stands 15 feet from the flagpole and holds a cardboard square, the edges of the square line up with the top and bottom of the flagpole. Approximate the height of the flagpole.

17. **HILLS** The length of a hill in your neighborhood is 2000 feet. The height of the hill is 750 feet. What is the angle of elevation of the hill?



xy ALGEBRA REVIEW

Animated Algebra classzone.com

GRAPH AND SOLVE QUADRATIC EQUATIONS

The graph of $y = ax^2 + bx + c$ is a parabola that opens upward if $a > 0$ and opens downward if $a < 0$. The x-coordinate of the vertex is $-\frac{b}{2a}$. The axis of symmetry is the vertical line $x = -\frac{b}{2a}$.

EXAMPLE 1 *Graph a quadratic function*

Graph the equation $y = -x^2 + 4x - 3$.

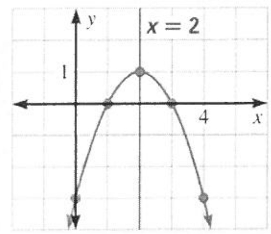
Because $a = -1$ and $-1 < 0$, the graph opens downward.

The vertex has x-coordinate $-\frac{b}{2a} = -\frac{4}{2(-1)} = 2$.

The y-coordinate of the vertex is $-(2)^2 + 4(2) - 3 = 1$.

So, the vertex is (2, 1) and the axis of symmetry is $x = 2$.

Use a table of values to draw a parabola through the plotted points.



EXAMPLE 2 *Solve a quadratic equation by graphing*

Solve the equation $x^2 - 2x = 3$.

Write the equation in the standard form $ax^2 + bx + c = 0$:

$x^2 - 2x - 3 = 0.$

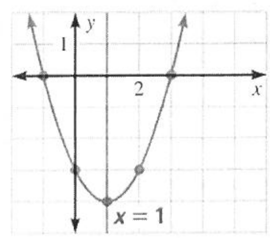
Graph the related quadratic function $y = x^2 - 2x - 3$, as shown.

The x-intercepts of the graph are -1 and 3.

So, the solutions of $x^2 - 2x = 3$ are -1 and 3.

Check the solution algebraically.

$(-1)^2 - 2(-1) \stackrel{?}{=} 3 \rightarrow 1 + 2 = 3$ $\quad$ $(3)^2 - 2(3) \stackrel{?}{=} 3 \rightarrow 9 - 6 = 3$ ✓



EXERCISES

EXAMPLE 1 for Exs. 1–6

Graph the quadratic function. Label the vertex and axis of symmetry.

1. $y = x^2 - 6x + 8$
2. $y = -x^2 - 4x + 2$
3. $y = 2x^2 - x - 1$
4. $y = 3x^2 - 9x + 2$
5. $y = \frac{1}{2}x^2 - x + 3$
6. $y = -4x^2 + 6x - 5$

EXAMPLE 2 for Exs. 7–18

Solve the quadratic equation by graphing. Check solutions algebraically.

7. $x^2 = x + 6$
8. $4x + 4 = -x^2$
9. $2x^2 = -8$
10. $3x^2 + 2 = 14$
11. $-x^2 + 4x - 5 = 0$
12. $2x - x^2 = -15$
13. $\frac{1}{4}x^2 = 2x$
14. $x^2 + 3x = 4$
15. $x^2 + 8 = 6x$
16. $x^2 = 9x - 1$
17. $-25 = x^2 + 10x$
18. $x^2 + 6x = 0$