7.5 Apply the Tangent Ratio

Before You used congruent or similar triangles for indirect measurement.

Now You will use the tangent ratio for indirect measurement.

Why? So you can find the height of a roller coaster, as in Ex. 32.

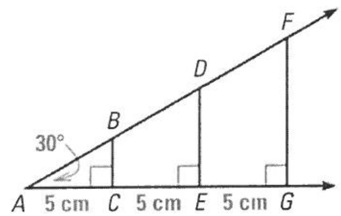
Key Vocabulary
- **trigonometric ratio**
- **tangent**

ACTIVITY RIGHT TRIANGLE RATIO

Materials: metric ruler, protractor, calculator

STEP 1 **Draw** a 30° angle and mark a point every 5 centimeters on a side as shown. Draw perpendicular segments through the 3 points.

STEP 2 **Measure** the legs of each right triangle. Copy and complete the table.

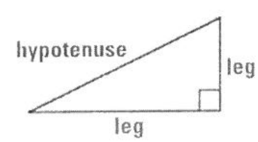


Triangle	Adjacent leg	Opposite leg	$\frac{\text{Opposite leg}}{\text{Adjacent leg}}$
$\triangle ABC$	5 cm	?	?
$\triangle ADE$	10 cm	?	?
$\triangle AFG$	15 cm	?	?

STEP 3 **Explain** why the proportions $\frac{BC}{DE} = \frac{AC}{AE}$ and $\frac{BC}{AC} = \frac{DE}{AE}$ are true.

STEP 4 **Make** a conjecture about the ratio of the lengths of the legs in a right triangle. Test your conjecture by using different acute angle measures.

A **trigonometric ratio** is a ratio of the lengths of two sides in a right triangle. You will use trigonometric ratios to find the measure of a side or an acute angle in a right triangle.



The ratio of the lengths of the legs in a right triangle is constant for a given angle measure. This ratio is called the **tangent** of the angle.

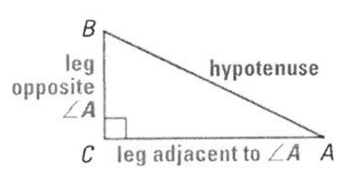
ABBREVIATE
Remember these abbreviations:
tangent → tan
opposite → opp.
adjacent → adj.

KEY CONCEPT *For Your Notebook*

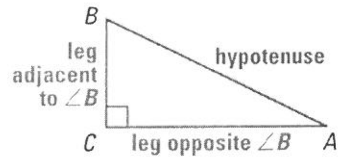
Tangent Ratio

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The tangent of $\angle A$ (written as $\tan A$) is defined as follows:

$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{BC}{AC}$$

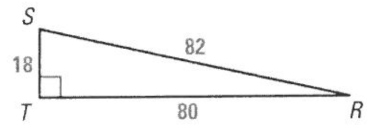


COMPLEMENTARY ANGLES In the right triangle, $\angle A$ and $\angle B$ are complementary so you can use the same diagram to find the tangent of $\angle A$ and the tangent of $\angle B$. Notice that the leg adjacent to $\angle A$ is the leg *opposite* $\angle B$ and the leg opposite $\angle A$ is the leg *adjacent* to $\angle B$.



EXAMPLE 1 Find tangent ratios

Find tan *S* and tan *R*. Write each answer as a fraction and as a decimal rounded to four places.



Solution

$$\tan S = \frac{\text{opp. } \angle S}{\text{adj. to } \angle S} = \frac{RT}{ST} = \frac{80}{18} = \frac{40}{9} \approx 4.4444$$

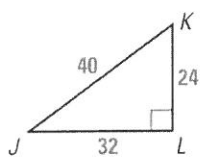
$$\tan R = \frac{\text{opp. } \angle R}{\text{adj. to } \angle R} = \frac{ST}{RT} = \frac{18}{80} = \frac{9}{40} = 0.2250$$

APPROXIMATE
Unless told otherwise, you should round the values of trigonometric ratios to the ten-thousandths' place and round lengths to the tenths' place.

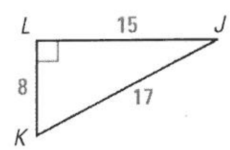
✓ **GUIDED PRACTICE** for Example 1

Find tan *J* and tan *K*. Round to four decimal places.

1.

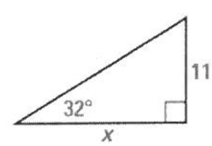


2.



EXAMPLE 2 Find a leg length

ALGEBRA **Find the value of x.**



Solution

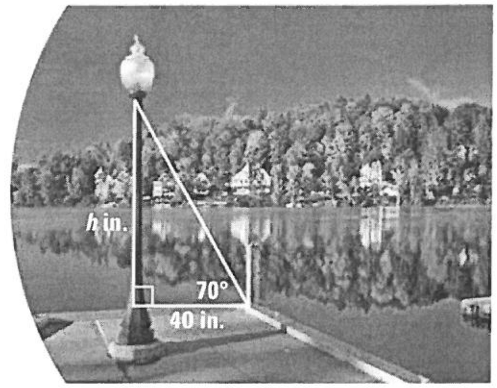
Use the tangent of an acute angle to find a leg length.

$\tan 32° = \frac{\text{opp.}}{\text{adj.}}$	Write ratio for tangent of 32°.
$\tan 32° = \frac{11}{x}$	Substitute.
$x \cdot \tan 32° = 11$	Multiply each side by x.
$x = \frac{11}{\tan 32°}$	Divide each side by tan 32°.
$x \approx \frac{11}{0.6249}$	Use a calculator to find tan 32°.
$x \approx 17.6$	Simplify.

ANOTHER WAY
You can also use the Table of Trigonometric Ratios on p. 925 to find the decimal values of trigonometric ratios.

EXAMPLE 3 Estimate height using tangent

LAMPPOST Find the height h of the lamppost to the nearest inch.



$\tan 70° = \frac{\text{opp.}}{\text{adj.}}$	Write ratio for tangent of 70°.
$\tan 70° = \frac{h}{40}$	Substitute.
$40 \cdot \tan 70° = h$	Multiply each side by 40.
$109.9 \approx h$	Use a calculator to simplify.

▶ The lamppost is about 110 inches tall.

SPECIAL RIGHT TRIANGLES You can find the tangent of an acute angle measuring 30°, 45°, or 60° by applying what you know about special right triangles.

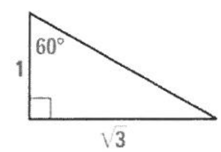
EXAMPLE 4 Use a special right triangle to find a tangent

Use a special right triangle to find the tangent of a 60° angle.

SIMILAR TRIANGLES
The tangents of all 60° angles are the same constant ratio. Any right triangle with a 60° angle can be used to determine this value.

STEP 1 **Because** all 30°-60°-90° triangles are similar, you can simplify your calculations by choosing 1 as the length of the shorter leg. Use the 30°-60°-90° Triangle Theorem to find the length of the longer leg.

longer leg = shorter leg $\cdot \sqrt{3}$	30°-60°-90° Triangle Theorem
$x = 1 \cdot \sqrt{3}$	Substitute.
$x = \sqrt{3}$	Simplify.



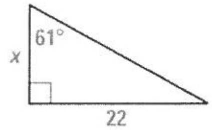
STEP 2 **Find** $\tan 60°$.

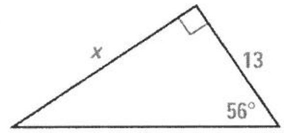
$\tan 60° = \frac{\text{opp.}}{\text{adj.}}$	Write ratio for tangent of 60°.
$\tan 60° = \frac{\sqrt{3}}{1}$	Substitute.
$\tan 60° = \sqrt{3}$	Simplify.

▶ The tangent of any 60° angle is $\sqrt{3} \approx 1.7321$.

✓ **GUIDED PRACTICE** for Examples 2, 3, and 4

Find the value of x. Round to the nearest tenth.

3. 

4. 

5. **WHAT IF?** In Example 4, suppose the side length of the shorter leg is 5 instead of 1. Show that the tangent of 60° is still equal to $\sqrt{3}$.

7.5 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 7, and 31
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 16, 17, 35, and 37

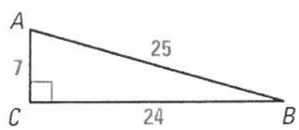
SKILL PRACTICE

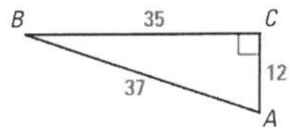
1. **VOCABULARY** Copy and complete: The tangent ratio compares the length of _?_ to the length of _?_.

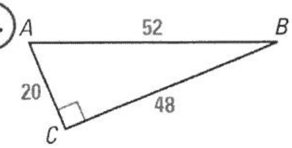
2. ★ **WRITING** *Explain* how you know that all right triangles with an acute angle measuring $n°$ are similar to each other.

EXAMPLE 1 on p. 467 for Exs. 3–5

FINDING TANGENT RATIOS **Find tan A and tan B. Write each answer as a fraction and as a decimal rounded to four places.**

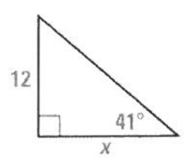
3. 

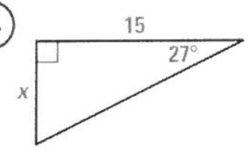
4. 

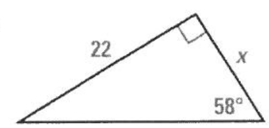
5. 

EXAMPLE 2 on p. 467 for Exs. 6–8

FINDING LEG LENGTHS **Find the value of x to the nearest tenth.**

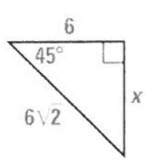
6. 

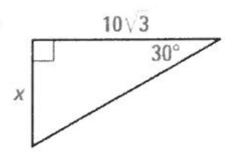
7. 

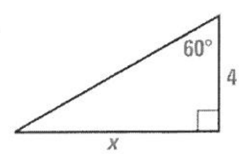
8. 

EXAMPLE 4 on p. 468 for Exs. 9–12

FINDING LEG LENGTHS **Find the value of x using the definition of tangent. Then find the value of x using the 45°-45°-90° Theorem or the 30°-60°-90° Theorem.** ***Compare*** **the results.**

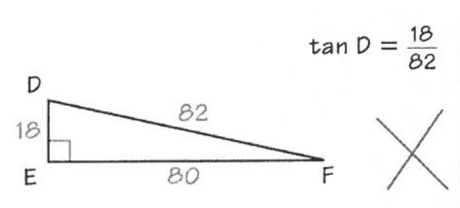
9. 

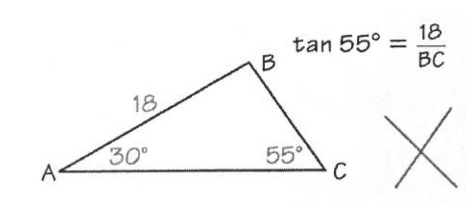
10. 

11. 

12. **SPECIAL RIGHT TRIANGLES** Find tan 30° and tan 45° using the 45°-45°-90° Triangle Theorem and the 30°-60°-90° Triangle Theorem.

ERROR ANALYSIS ***Describe*** **the error in the statement of the tangent ratio. Correct the statement, if possible. Otherwise, write** ***not possible.***

13. 

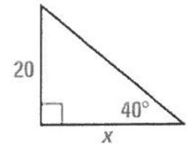
14. 

15. ★ **WRITING** *Describe* what you must know about a triangle in order to use the tangent ratio.

16. ★ **MULTIPLE CHOICE** Which expression can be used to find the value of x in the triangle shown?

Ⓐ $x = 20 \cdot \tan 40°$ Ⓑ $x = \frac{\tan 40°}{20}$

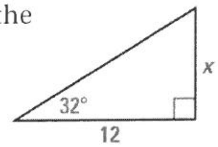
Ⓒ $x = \frac{20}{\tan 40°}$ Ⓓ $x = \frac{20}{\tan 50°}$



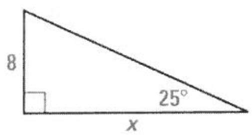
17. ★ **MULTIPLE CHOICE** What is the approximate value of x in the triangle shown?

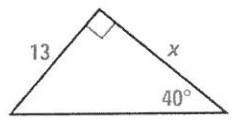
Ⓐ 0.4   Ⓑ 2.7

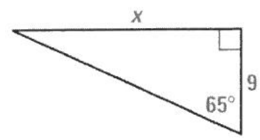
Ⓒ 7.5   Ⓓ 19.2



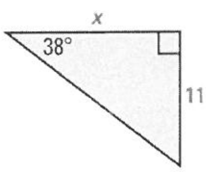
FINDING LEG LENGTHS **Use a tangent ratio to find the value of x. Round to the nearest tenth. Check your solution using the tangent of the other acute angle.**

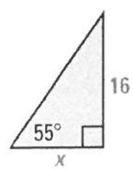
18. 

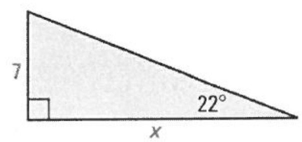
19. 

20. 

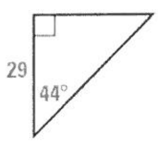
FINDING AREA **Find the area of the triangle. Round to the nearest tenth.**

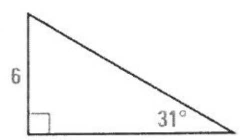
21. 

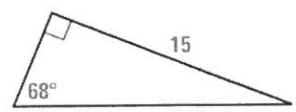
22. 

23. 

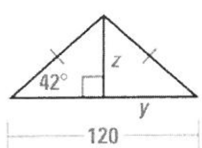
FINDING PERIMETER **Find the perimeter of the triangle. Round to the nearest tenth.**

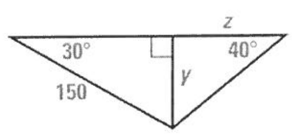
24. 

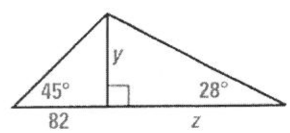
25. 

26. 

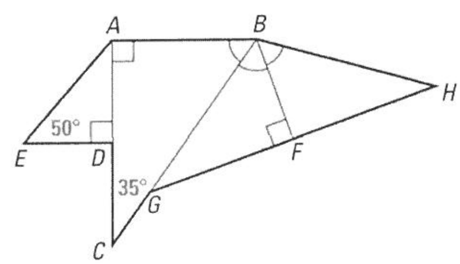
FINDING LENGTHS **Find y. Then find z. Round to the nearest tenth.**

27. 

28. 

29. 

30. **CHALLENGE** Find the perimeter of the figure at the right, where $AC = 26$, $AD = BF$, and D is the midpoint of $\overline{AC}$.



◯ = **WORKED-OUT SOLUTIONS on p. WS1**   ★ = **STANDARDIZED TEST PRACTICE**

PROBLEM SOLVING

EXAMPLE 3 on p. 468 for Exs. 31–32

31. **WASHINGTON MONUMENT** A surveyor is standing 118 feet from the base of the Washington Monument. The surveyor measures the angle between the ground and the top of the monument to be 78°. Find the height h of the Washington Monument to the nearest foot.

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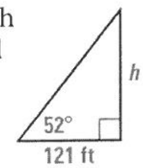
32. **ROLLER COASTERS** A roller coaster makes an angle of 52° with the ground. The horizontal distance from the crest of the hill to the bottom of the hill is about 121 feet, as shown. Find the height h of the roller coaster to the nearest foot.



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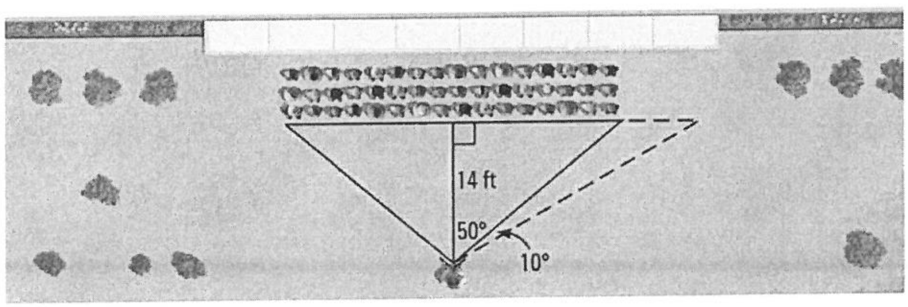
CLASS PICTURE Use this information and diagram for Exercises 33 and 34.

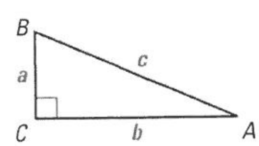
Your class is having a class picture taken on the lawn. The photographer is positioned 14 feet away from the center of the class. If she looks toward either end of the class, she turns 50°.



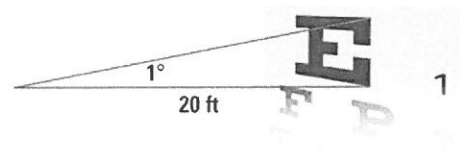
33. **ISOSCELES TRIANGLE** What is the distance between the ends of the class?

34. **MULTI-STEP PROBLEM** The photographer wants to estimate how many more students can fit at the end of the first row. The photographer turns 50° to see the last student and another 10° to see the end of the camera range.
    a. Find the distance from the center to the last student in the row.
    b. Find the distance from the center to the end of the camera range.
    c. Use the results of parts (a) and (b) to estimate the length of the empty space.
    d. If each student needs 2 feet of space, about how many more students can fit at the end of the first row? *Explain* your reasoning.

35. ★ **SHORT RESPONSE** Write expressions for the tangent of each acute angle in the triangle. *Explain* how the tangent of one acute angle is related to the tangent of the other acute angle. What kind of angle pair are $\angle A$ and $\angle B$?

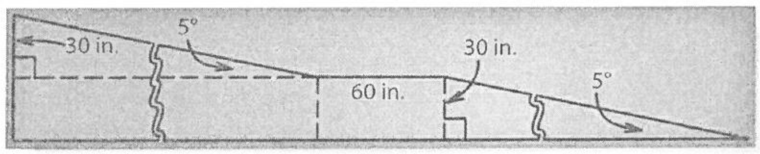


36. **EYE CHART** You are looking at an eye chart that is 20 feet away. Your eyes are level with the bottom of the "E" on the chart. To see the top of the "E," you look up 1°. How tall is the "E"?



Not drawn to scale

37. ★ **EXTENDED RESPONSE** According to the Americans with Disabilities Act, a ramp cannot have an incline that is greater than 5°. The regulations also state that the maximum rise of a ramp is 30 inches. When a ramp needs to reach a height greater than 30 inches, a series of ramps connected by 60 inch landings can be used, as shown below.

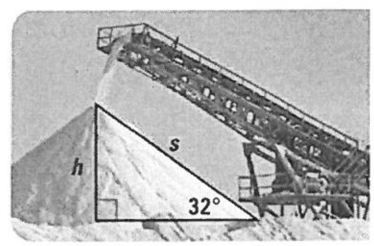


   a. What is the maximum horizontal length of the base of one ramp, in feet? Round to the nearest foot.

   b. If a doorway is 7.5 feet above the ground, what is the least number of ramps and landings you will need to lead to the doorway? Draw and label a diagram to *justify* your answer.

   c. To the nearest foot, what is the total length of the base of the system of ramps and landings in part (b)?

38. **CHALLENGE** The road salt shown is stored in a cone-shaped pile. The base of the cone has a circumference of 80 feet. The cone rises at an angle of 32°. Find the height h of the cone. Then find the length s of the cone-shaped pile.



MIXED REVIEW

The expressions given represent the angle measures of a triangle. Find the measure of each angle. Then classify the triangle by its angles. *(p. 217)*

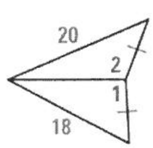
39. $m\angle A = x^\circ$
$m\angle B = 4x^\circ$
$m\angle C = 4x^\circ$

40. $m\angle A = x^\circ$
$m\angle B = x^\circ$
$m\angle C = (5x - 60)^\circ$

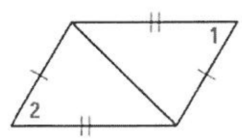
41. $m\angle A = (x + 20)^\circ$
$m\angle B = (3x + 15)^\circ$
$m\angle C = (x - 30)^\circ$

Copy and complete the statement with <, >, or =. *Explain.* *(p. 335)*

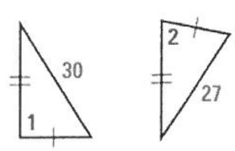
42. $m\angle 1$ _?_ $m\angle 2$



43. $m\angle 1$ _?_ $m\angle 2$



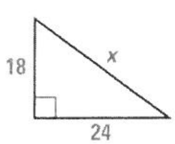
44. $m\angle 1$ _?_ $m\angle 2$



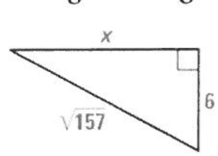
PREVIEW
Prepare for Lesson 7.6 in Exs. 45–47.

Find the unknown side length of the right triangle. *(p. 433)*

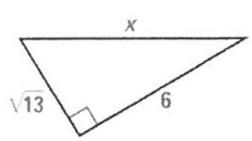
45.



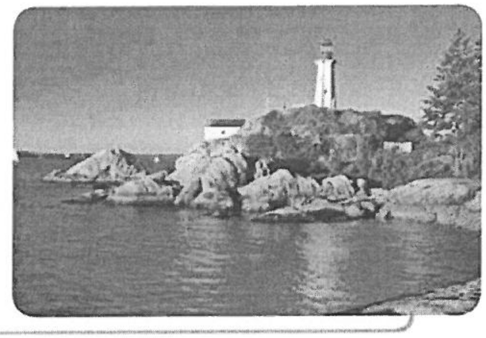
46.



47.



7.6 Apply the Sine and Cosine Ratios



Before You used the tangent ratio.

Now You will use the sine and cosine ratios.

Why So you can find distances, as in Ex. 39.

Key Vocabulary
- **sine**
- **cosine**
- **angle of elevation**
- **angle of depression**

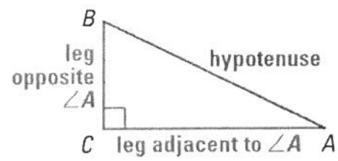
The **sine** and **cosine** ratios are trigonometric ratios for acute angles that involve the lengths of a leg and the hypotenuse of a right triangle.

ABBREVIATE
Remember these abbreviations:
sine → sin
cosine → cos
hypotenuse → hyp

KEY CONCEPT *For Your Notebook*

Sine and Cosine Ratios

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The sine of $\angle A$ and cosine of $\angle A$ (written $\sin A$ and $\cos A$) are defined as follows:

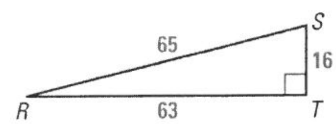


$$\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{BC}{AB}$$

$$\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{AC}{AB}$$

EXAMPLE 1 Find sine ratios

Find $\sin S$ and $\sin R$. Write each answer as a fraction and as a decimal rounded to four places.



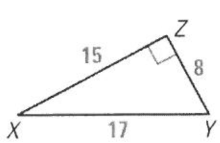
Solution

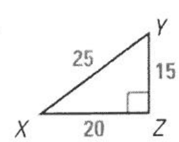
$$\sin S = \frac{\text{opp. } \angle S}{\text{hyp.}} = \frac{RT}{SR} = \frac{63}{65} \approx 0.9692$$

$$\sin R = \frac{\text{opp. } \angle R}{\text{hyp.}} = \frac{ST}{SR} = \frac{16}{65} \approx 0.2462$$

✓ **GUIDED PRACTICE** for Example 1

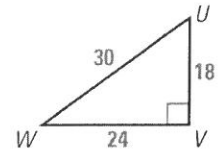
Find $\sin X$ and $\sin Y$. Write each answer as a fraction and as a decimal. Round to four decimal places, if necessary.

1.


2.


EXAMPLE 2 Find cosine ratios

Find cos U and cos W. Write each answer as a fraction and as a decimal.



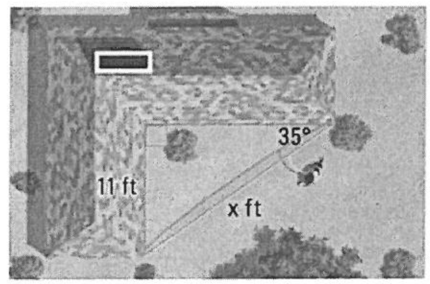
Solution

$$\cos U = \frac{\text{adj. to } \angle U}{\text{hyp.}} = \frac{UV}{UW} = \frac{18}{30} = \frac{3}{5} = 0.6000$$

$$\cos W = \frac{\text{adj. to } \angle W}{\text{hyp.}} = \frac{WV}{UW} = \frac{24}{30} = \frac{4}{5} = 0.8000$$

EXAMPLE 3 Use a trigonometric ratio to find a hypotenuse

DOG RUN You want to string cable to make a dog run from two corners of a building, as shown in the diagram. Write and solve a proportion using a trigonometric ratio to approximate the length of cable you will need.



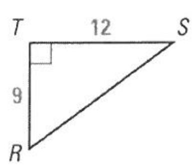
Solution

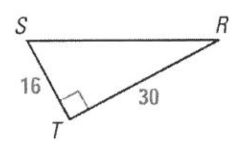
$\sin 35° = \frac{\text{opp.}}{\text{hyp.}}$	Write ratio for sine of 35°.
$\sin 35° = \frac{11}{x}$	Substitute.
$x \cdot \sin 35° = 11$	Multiply each side by x.
$x = \frac{11}{\sin 35°}$	Divide each side by sin 35°.
$x \approx \frac{11}{0.5736}$	Use a calculator to find sin 35°.
$x \approx 19.2$	Simplify.

▶ You will need a little more than 19 feet of cable.

✓ **GUIDED PRACTICE** for Examples 2 and 3

In Exercises 3 and 4, find cos R and cos S. Write each answer as a decimal. Round to four decimal places, if necessary.

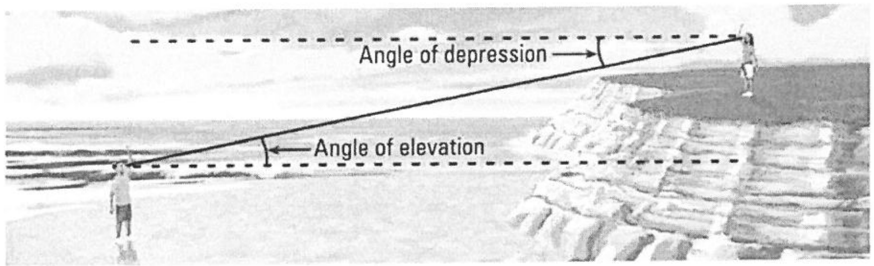
3. 

4. 

5. In Example 3, use the cosine ratio to find the length of the other leg of the triangle formed.

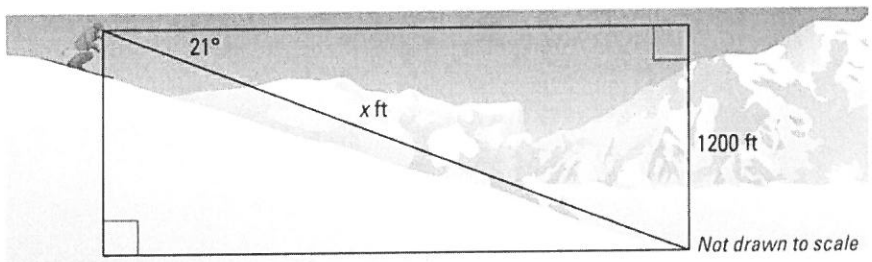
ANGLES If you look up at an object, the angle your line of sight makes with a horizontal line is called the **angle of elevation**. If you look down at an object, the angle your line of sight makes with a horizontal line is called the **angle of depression**.

APPLY THEOREMS
Notice that the angle of elevation and the angle of depression are congruent by the Alternate Interior Angles Theorem on page 155.



EXAMPLE 4 Find a hypotenuse using an angle of depression

SKIING You are skiing on a mountain with an altitude of 1200 meters. The angle of depression is 21°. About how far do you ski down the mountain?



Solution

$\sin 21° = \frac{\text{opp.}}{\text{hyp.}}$ Write ratio for sine of 21°.

$\sin 21° = \frac{1200}{x}$ Substitute.

$x \cdot \sin 21° = 1200$ Multiply each side by x.

$x = \frac{1200}{\sin 21°}$ Divide each side by sin 21°.

$x \approx \frac{1200}{0.3584}$ Use a calculator to find sin 21°.

$x \approx 3348.2$ Simplify.

▸ You ski about 3348 meters down the mountain.

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✓ **GUIDED PRACTICE** for Example 4

6. **WHAT IF?** Suppose the angle of depression in Example 4 is 28°. About how far would you ski?

EXAMPLE 5 Find leg lengths using an angle of elevation

SKATEBOARD RAMP You want to build a skateboard ramp with a length of 14 feet and an angle of elevation of 26°. You need to find the height and length of the base of the ramp.

ANOTHER WAY
For alternative methods for solving the problem in Example 5, turn to page 481 for the **Problem Solving Workshop**.

Solution

STEP 1 **Find** the height.

$\sin 26° = \frac{\text{opp.}}{\text{hyp.}}$ Write ratio for sine of 26°.

$\sin 26° = \frac{x}{14}$ Substitute.

$14 \cdot \sin 26° = x$ Multiply each side by 14.

$6.1 \approx x$ Use a calculator to simplify.

▶ The height is about 6.1 feet.

STEP 2 **Find** the length of the base.

$\cos 26° = \frac{\text{adj.}}{\text{hyp.}}$ Write ratio for cosine of 26°.

$\cos 26° = \frac{y}{14}$ Substitute.

$14 \cdot \cos 26° = y$ Multiply each side by 14.

$12.6 \approx y$ Use a calculator to simplify.

▶ The length of the base is about 12.6 feet.

EXAMPLE 6 Use a special right triangle to find a sine and cosine

Use a special right triangle to find the sine and cosine of a 60° angle.

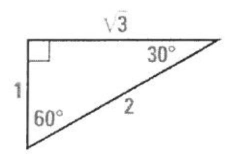
DRAW DIAGRAMS
As in Example 4 on page 468, to simplify calculations you can choose 1 as the length of the shorter leg.

Solution

Use the 30°-60°-90° Triangle Theorem to draw a right triangle with side lengths of 1, $\sqrt{3}$, and 2. Then set up sine and cosine ratios for the 60° angle.

$$\sin 60° = \frac{\text{opp.}}{\text{hyp.}} = \frac{\sqrt{3}}{2} \approx 0.8660$$

$$\cos 60° = \frac{\text{adj.}}{\text{hyp.}} = \frac{1}{2} = 0.5000$$



✓ **GUIDED PRACTICE** for Examples 5 and 6

7. **WHAT IF?** In Example 5, suppose the angle of elevation is 35°. What is the new height and base length of the ramp?

8. Use a special right triangle to find the sine and cosine of a 30° angle.

7.6 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 9, and 33

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 17, 18, 29, 35, and 37

◆ = **MULTIPLE REPRESENTATIONS** Ex. 39

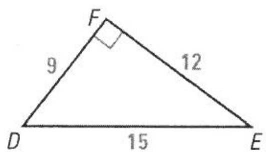
SKILL PRACTICE

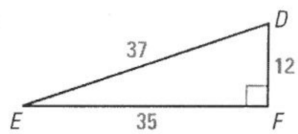
1. **VOCABULARY** Copy and complete: The sine ratio compares the length of _?_ to the length of _?_.

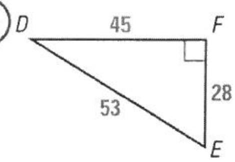
2. ★ **WRITING** *Explain* how to tell which side of a right triangle is adjacent to an angle and which side is the hypotenuse.

EXAMPLE 1 on p. 473 for Exs. 3–6

FINDING SINE RATIOS Find sin *D* and sin *E*. Write each answer as a fraction and as a decimal. Round to four decimal places, if necessary.

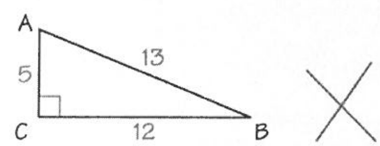
3. 

4. 

5. 

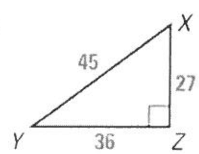
6. **ERROR ANALYSIS** *Explain* why the student's statement is incorrect. Write a correct statement for the sine of the angle.

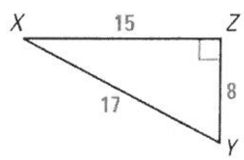
$\sin A = \frac{5}{13}$

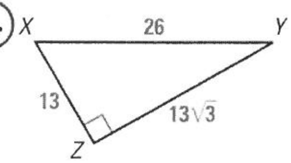


EXAMPLE 2 on p. 474 for Exs. 7–9

FINDING COSINE RATIOS Find cos *X* and cos *Y*. Write each answer as a fraction and as a decimal. Round to four decimal places, if necessary.

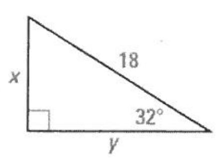
7. 

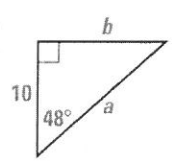
8. 

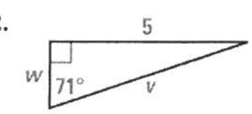
9. 

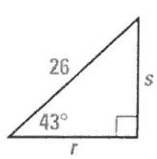
EXAMPLE 3 on p. 474 for Exs. 10–15

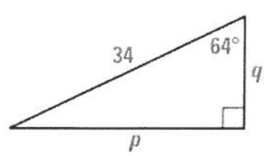
USING SINE AND COSINE RATIOS Use a sine or cosine ratio to find the value of each variable. Round decimals to the nearest tenth.

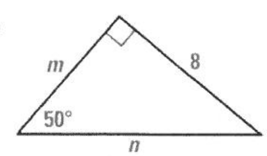
10. 

11. 

12. 

13. 

14. 

15. 

EXAMPLE 6 on p. 476 for Ex. 16

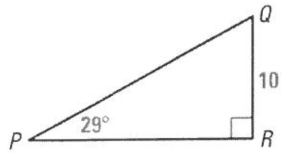
16. **SPECIAL RIGHT TRIANGLES** Use the 45°-45°-90° Triangle Theorem to find the sine and cosine of a 45° angle.

17. ★ **WRITING** *Describe* what you must know about a triangle in order to use the sine ratio and the cosine ratio.

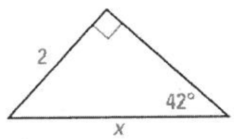
18. ★ **MULTIPLE CHOICE** In $\triangle PQR$, which expression can be used to find PQ?

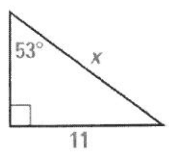
(A) $10 \cdot \cos 29°$ (B) $10 \cdot \sin 29°$

(C) $\frac{10}{\sin 29°}$ (D) $\frac{10}{\cos 29°}$



ALGEBRA Find the value of x. Round decimals to the nearest tenth.

19.


20.


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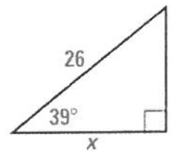

FINDING SINE AND COSINE RATIOS Find the unknown side length. Then find sin X and cos X. Write each answer as a fraction in simplest form and as a decimal. Round to four decimal places, if necessary.

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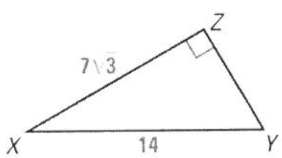

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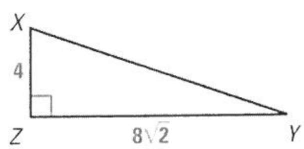

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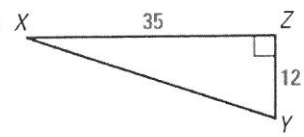

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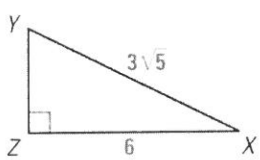

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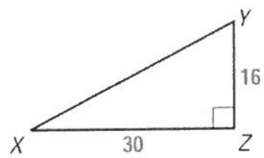

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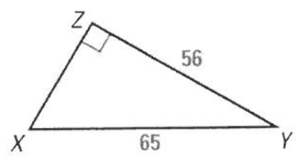


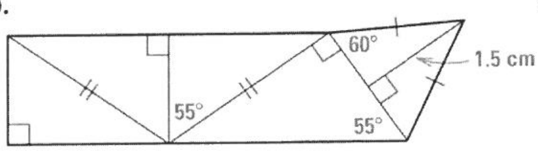
28. **ANGLE MEASURE** Make a prediction about how you could use trigonometric ratios to find angle measures in a triangle.

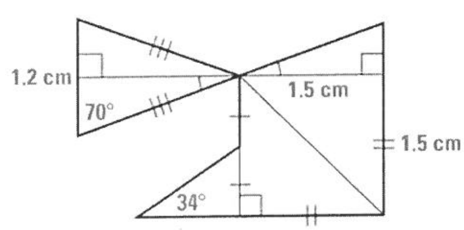
29. ★ **MULTIPLE CHOICE** In $\triangle JKL$, $m\angle L = 90°$. Which statement about $\triangle JKL$ *cannot* be true?

(A) $\sin J = 0.5$ (B) $\sin J = 0.1071$

(C) $\sin J = 0.8660$ (D) $\sin J = 1.1$

PERIMETER Find the approximate perimeter of the figure.

30.


31.


32. **CHALLENGE** Let A be any acute angle of a right triangle. Show that (a) $\tan A = \frac{\sin A}{\cos A}$ and (b) $(\sin A)^2 + (\cos A)^2 = 1$.

◯ = **WORKED-OUT SOLUTIONS on p. WS1** ★ = **STANDARDIZED TEST PRACTICE**

PROBLEM SOLVING

EXAMPLES 4 and 5 on pp. 475–476 for Exs. 33–36

33. AIRPLANE RAMP The airplane door is 19 feet off the ground and the ramp has a 31° angle of elevation. What is the length y of the ramp?

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34. BLEACHERS Find the horizontal distance h the bleachers cover. Round to the nearest foot.

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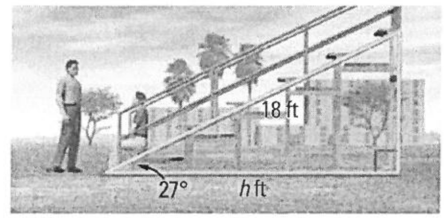
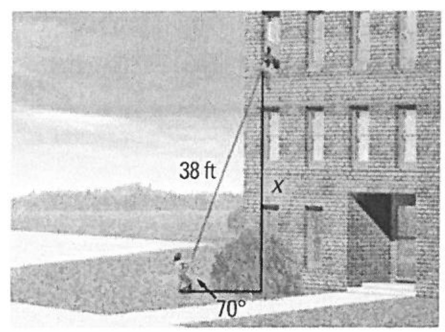


35. ★ SHORT RESPONSE You are flying a kite with 20 feet of string extended. The angle of elevation from the spool of string to the kite is 41°.

a. Draw and label a diagram to represent the situation.

b. How far off the ground is the kite if you hold the spool 5 feet off the ground? *Describe* how the height where you hold the spool affects the height of the kite.

36. MULTI-STEP PROBLEM You want to hang a banner that is 29 feet tall from the third floor of your school. You need to know how tall the wall is, but there is a large bush in your way.

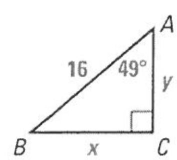


a. You throw a 38 foot rope out of the window to your friend. She extends it to the end and measures the angle of elevation to be 70°. How high is the window?

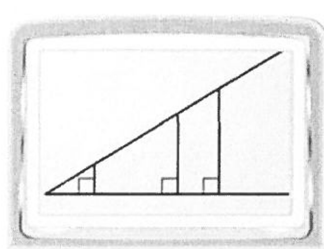
b. The bush is 6 feet tall. Will your banner fit above the bush?

c. **What If?** Suppose you need to find how far from the school your friend needs to stand. Which trigonometric ratio should you use?

37. ★ SHORT RESPONSE Nick uses the equation $\sin 49° = \frac{x}{16}$ to find BC in $\triangle ABC$. Tim uses the equation $\cos 41° = \frac{x}{16}$. Which equation produces the correct answer? *Explain.*



38. TECHNOLOGY Use geometry drawing software to construct an angle. Mark three points on one side of the angle and construct segments perpendicular to that side at the points. Measure the legs of each triangle and calculate the sine of the angle. Is the sine the same for each triangle?



39. **MULTIPLE REPRESENTATIONS** You are standing on a cliff 30 feet above an ocean. You see a sailboat on the ocean.

a. **Drawing a Diagram** Draw and label a diagram of the situation.

b. **Making a Table** Make a table showing the angle of depression and the length of your line of sight. Use the angles 40°, 50°, 60°, 70°, and 80°.

c. **Drawing a Graph** Graph the values you found in part (b), with the angle measures on the x-axis.

d. **Making a Prediction** Predict the length of the line of sight when the angle of depression is 30°.

40. **ALGEBRA** If $\triangle EQU$ is equilateral and $\triangle RGT$ is a right triangle with $RG = 2$, $RT = 1$, and $m\angle T = 90°$, show that $\sin E = \cos G$.

41. **CHALLENGE** Make a conjecture about the relationship between sine and cosine values.

a. Make a table that gives the sine and cosine values for the acute angles of a 45°-45°-90° triangle, a 30°-60°-90° triangle, a 34°-56°-90° triangle, and a 17°-73°-90° triangle.

b. Compare the sine and cosine values. What pattern(s) do you notice?

c. Make a conjecture about the sine and cosine values in part (b).

d. Is the conjecture in part (c) true for right triangles that are not special right triangles? *Explain.*

MIXED REVIEW

Rewrite the equation so that x is a function of y. *(p. 877)*

42. $y = \sqrt{x}$

43. $y = 3x - 10$

44. $y = \frac{x}{9}$

PREVIEW
Prepare for Lesson 7.7 in Exs. 45–47.

Copy and complete the table. *(p. 884)*

45.

x	$\sqrt{x}$
?	0
?	1
?	$\sqrt{2}$
?	2
?	4

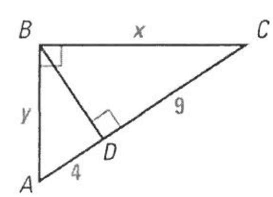
46.

x	$\frac{1}{x}$
?	1
?	$\frac{1}{2}$
?	3
?	$\frac{2}{7}$
?	7

47.

x	$\frac{2}{7}x + 4$
?	0
?	2
?	6
?	8
?	10

48. Find the values of x and y in the triangle at the right. *(p. 449)*



PROBLEM SOLVING WORKSHOP LESSON 7.6

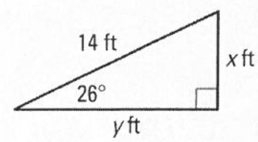
Using ALTERNATIVE METHODS

Another Way to Solve Example 5, page 476

MULTIPLE REPRESENTATIONS You can use the Pythagorean Theorem, tangent ratio, sine ratio, or cosine ratio to find the length of an unknown side of a right triangle. The decision of which method to use depends upon what information you have. In some cases, you can use more than one method to find the unknown length.

PROBLEM

SKATEBOARD RAMP You want to build a skateboard ramp with a length of 14 feet and an angle of elevation of 26°. You need to find the height and base of the ramp.



METHOD 1

Using a Cosine Ratio and the Pythagorean Theorem

STEP 1 **Find** the measure of the third angle.

$26° + 90° + m\angle 3 = 180°$	Triangle Sum Theorem
$116° + m\angle 3 = 180°$	Combine like terms.
$m\angle 3 = 64°$	Subtact 116° from each side.

STEP 2 **Use** the cosine ratio to find the height of the ramp.

$\cos 64° = \frac{\text{adj.}}{\text{hyp.}}$	Write ratio for cosine of 64°.
$\cos 64° = \frac{x}{14}$	Substitute.
$14 \cdot \cos 64° = x$	Multiply each side by 14.
$6.1 \approx x$	Use a calculator to simplify.

▸ The height is about 6.1 feet.

STEP 3 **Use** the Pythagorean Theorem to find the length of the base of the ramp.

$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$	Pythagorean Theorem
$14^2 = 6.1^2 + y^2$	Substitute.
$196 = 37.21 + y^2$	Multiply.
$158.79 = y^2$	Subtract 37.21 from each side.
$12.6 \approx y$	Find the positive square root.

▸ The length of the base is about 12.6 feet.

METHOD 2 Using a Tangent Ratio

Use the tangent ratio and $h = 6.1$ feet to find the length of the base of the ramp.

$\tan 26° = \frac{\text{opp.}}{\text{adj.}}$	Write ratio for tangent of 26°.
$\tan 26° = \frac{6.1}{y}$	Substitute.
$y \cdot \tan 26° = 61$	Multiply each side by y.
$y = \frac{6.1}{\tan 26°}$	Divide each side by tan 26°.
$y \approx 12.5$	Use a calculator to simplify.

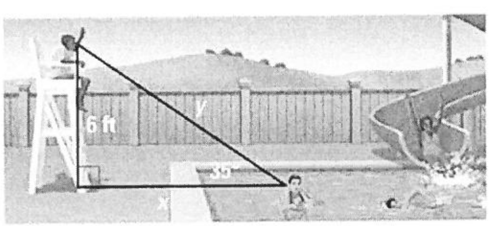
▸ The length of the base is about 12.5 feet.

Notice that when using the Pythagorean Theorem, the length of the base is 12.6 feet, but when using the tangent ratio, the length of the base is 12.5 feet. The tenth of a foot difference is due to the rounding error introduced when finding the height of the ramp and using that rounded value to calculate the length of the base.

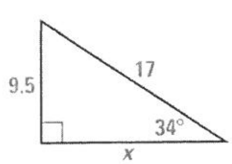
PRACTICE

1. **WHAT IF?** Suppose the length of the skateboard ramp is 20 feet. Find the height and base of the ramp.

2. **SWIMMER** The angle of elevation from the swimmer to the lifeguard is 35°. Find the distance x from the swimmer to the base of the lifeguard chair. Find the distance y from the swimmer to the lifeguard.

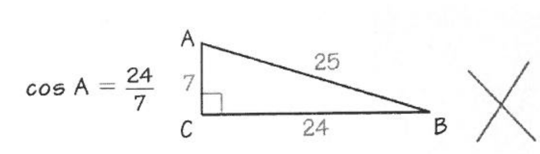


3. **ALGEBRA** Use the triangle below to write three different equations you can use to find the unknown leg length.



4. **SHORT RESPONSE** *Describe* how you would decide whether to use the Pythagorean Theorem or trigonometric ratios to find the lengths of unknown sides of a right triangle.

5. **ERROR ANALYSIS** *Explain* why the student's statement is incorrect. Write a correct statement for the cosine of the angle.



6. **EXTENDED RESPONSE** You want to find the height of a tree in your yard. The tree's shadow is 15 feet long and you measure the angle of elevation from the end of the shadow to the top of tree to be 75°.
   a. Find the height of the tree. *Explain* the method you chose to solve the problem.
   b. What else would you need to know to solve this problem using similar triangles.
   c. *Explain* why you cannot use the sine ratio to find the height of the tree.