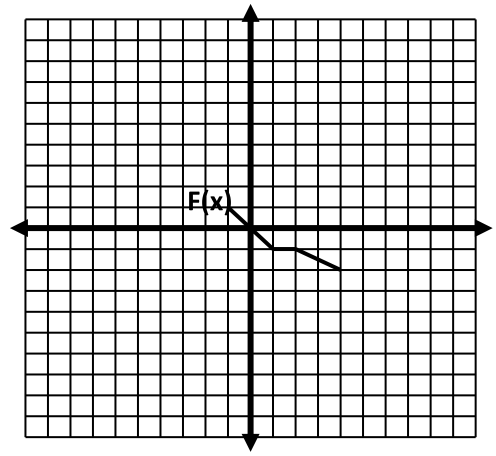
Transformations with Fred Functions

I. To the right is a graph of a **"Fred" function**. We can use Fred functions to explore transformations in the coordinate plane. **How do we know that Fred is a function?



1. Using the graph, what is the domain of Fred?

2. Using the graph, what is the range of Fred?

Let's explore the points on Fred.

3. How many points lie on Fred? Can you list them all?

4. What are the key points that would help us graph Fred?

II. We are going to call these key points **"characteristic" points**. It is important when graphing a function that you are able to identify these characteristic points. Use the graph of graph to evaluate the following.

F(1) = ______ F(–1) = ______ F(______) = –2 F(5) = _______

5. Remember that F(x) is another name for the y-values.

   Therefore the equation of Fred is **y = F(x)**.

x	F(x)
–1	
1	
2	
4	

6. Now let's try graphing Freddie Jr.: **y = F(x) + 4**. Complete the table below for this new function and then graph Freddie Jr. on the coordinate plane above.

y = F(x) + 4

x	y
–1	
1	
2	
4	

7. What type of transformation maps Fred, F(x), to Freddie Jr., F(x) + 4? (Be specific.)

8. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

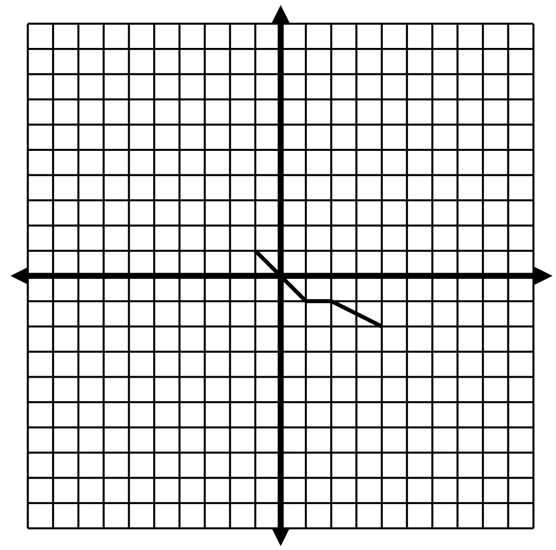
9. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

10. In y = F(x) + 4, how did the "+4" affect the graph of Fred? Did it affect the domain or the range?

Suppose Freddie Jr's equation is: **y = F(x) – 3**. Complete the table below for this new function and then graph Freddie Jr. on the coordinate plane above.

y = F(x) – 3

x	y
–1	
1	
2	
4	



11. What type of transformation maps Fred, F(x), to Freddie Jr., F(x) – 3? Be specific.

12. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

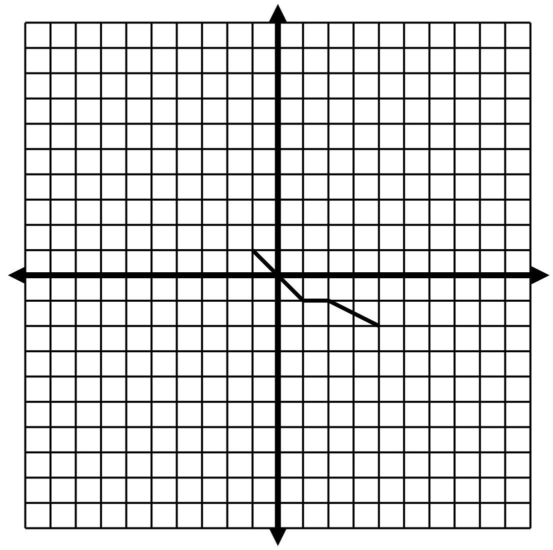
13. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

14. In y = F(x) – 3, how did the "– 3" affect the graph of Fred? Did it affect the domain or the range?

**Checkpoint: Using the understanding you have gained so far, describe the affect to Fred for the following functions.

Equation	**Effect to Fred's graph**
Example: y=F(x) + 18	Translate up 18 units
1. y = F(x) – 100	
2. y = F(x) + 73	
3. y = F(x) + 32	
4. y = F(x) – 521	

Suppose Freddie Jr's equation is: **y = F(x + 4)**.



15. Complete the table.

x	x + 4	y
–5	–1	1
	1	–1
	2	–1
	4	–2

*(**Hint**: Since, x + 4 = –1, subtract 4 from both sides of the equation, and x = –5. Use a similar method to find the missing x values.)*

16. On the coordinate plane above, graph the 4 ordered pairs (x, y). The first point is (–5, 1).

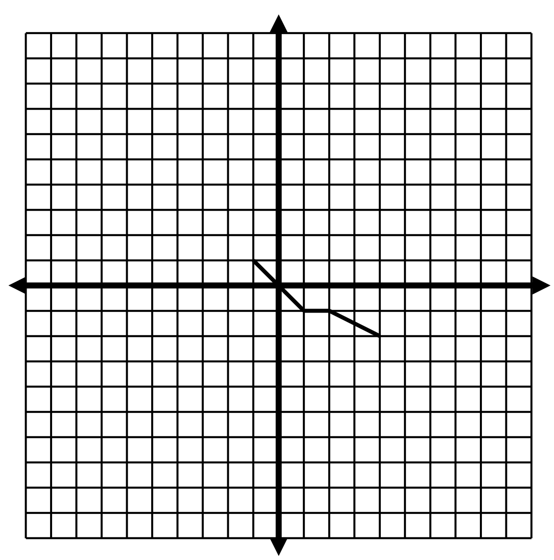
17. What type of transformation maps Fred, F(x), to Freddie Jr., F(x + 4)? (Be specific.)

18. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

19. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

20. In y = F(x + 4), how did the "+4" affect the graph of Fred? Did it affect the domain or the range?

Suppose Freddie Jr's equation is: **y = F(x – 3)**. Complete the table below for this new function and then graph Freddie Jr. on the coordinate plane above.



21. Complete the table.

y = F(x – 3)

x	x – 3	y
	–1	
	1	
	2	
	4	

22. On the coordinate plane at left, graph the 4 ordered pairs (x, y). [*Hint: The 1st point should be (2, 1).*]

23. What type of transformation maps Fred, F(x), to Freddie Jr., F(x – 3)? (Be specific.)

24. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

25. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

26. In y = F(x – 3), how did the " –3" affect the graph of Fred? Did it affect the domain or the range?

**Checkpoint: Using the understanding you have gained so far, describe the effect to Fred for the following functions.

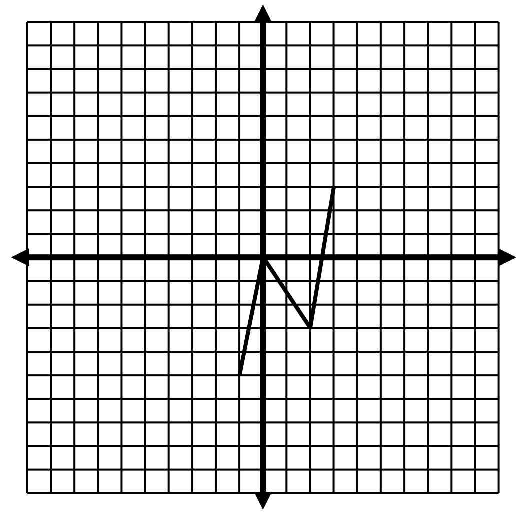
Equation	**Effect to Fred's graph**
Example: y=F(x + 18)	Translate left 18 units
1. y = F(x – 10)	
2. y = F(x) + 7	
3. y = F(x + 48)	
4. y = F(x) – 22	
5. y = F(x + 30) + 18	

**Checkpoint: Using the understanding you have gained so far, write the equation that would have the following effect on Fred's graph.

Equation	**Effect to Fred's graph**
Example: y=F(x + 8)	Translate left 8 units
1.	Translate up 29 units
2.	Translate right 7
3.	Translate left 45
4.	Translate left 5 and up 14
5.	Translate down 2 and right 6

Now let's look at a new function.

Its notation is **H(x)**, and we will call it **Harry**.
Use Harry to demonstrate what you have learned so far about the transformations of functions.



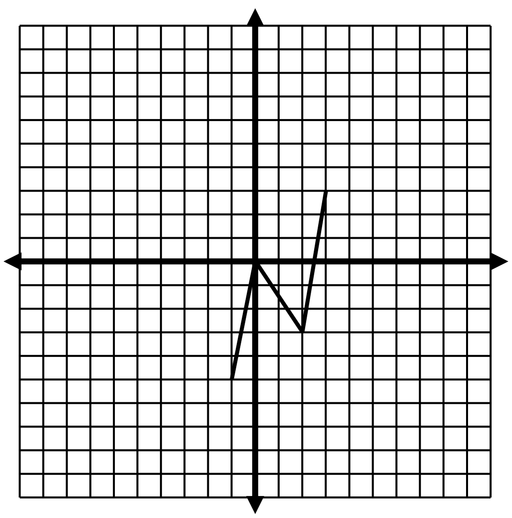
27. What are Harry's characteristic points?

____

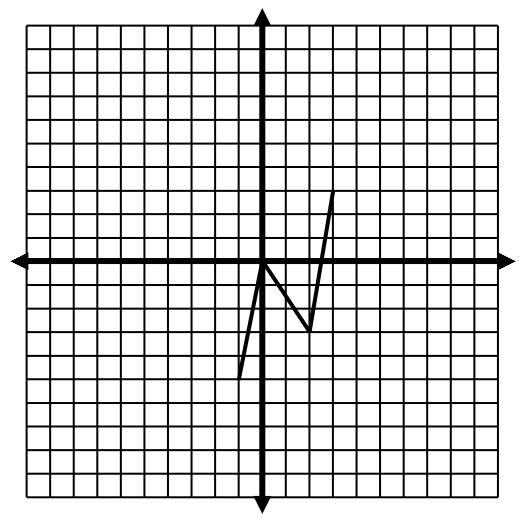
28. Describe the effect on Harry's graph for each of the following.
    a. H(x – 2) ____
    b. H(x) + 7 ____
    c. H(x+2) – 3 ____

29. Use your answers to questions 1 and 2 to help you sketch each graph *without using a table*.

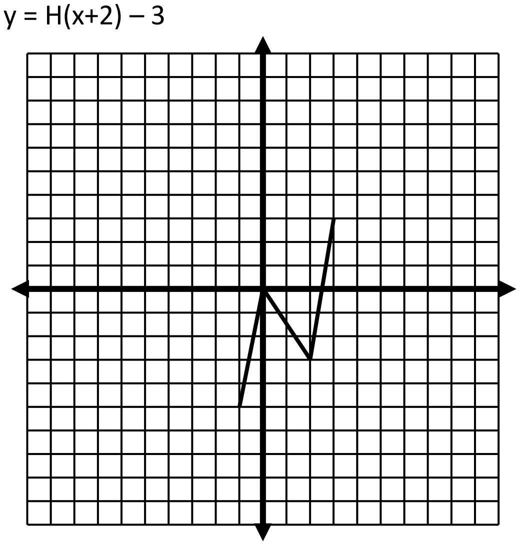
a. y = H(x – 2)



b. y = H(x) + 7



c. y = H(x+2) – 3

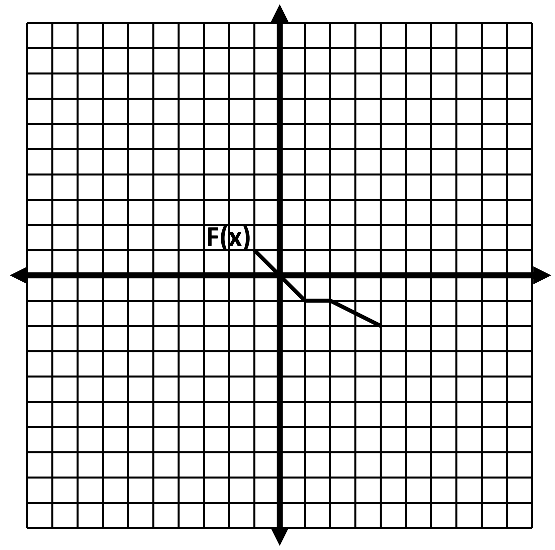


Let’s investigate transformations other than translations.

Recall that the equation for Fred is **y = F(x)**.

Complete the chart with Fred’s characteristic points.

x	F(x)



30. Let’s suppose that Freddie Jr. is **y = – F(x)**
Complete the table.

y = – F(x)

x	F(x)	y
–1	1	–1
1		
2		
4		

31. On the coordinate plane above, graph the 4 ordered pairs (x, y). [*Hint: The 1st point should be (–1, –1).*]

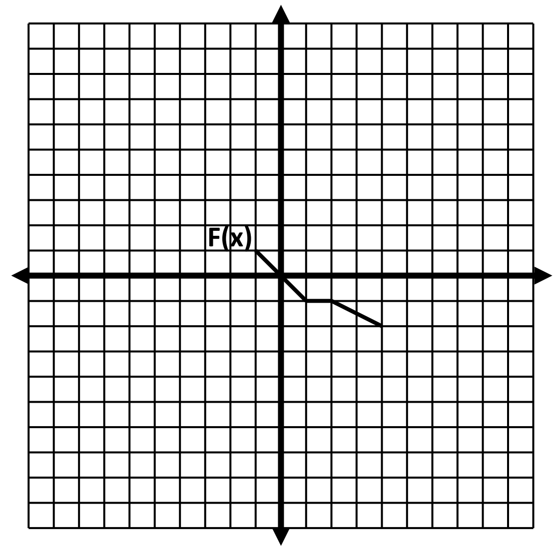
32. What type of transformation maps Fred, F(x), to Freddie Jr., –F(x)? (Be specific.)

33. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

34. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

35. In y = – F(x), how did the negative coefficient of “F(x)” affect the graph of Fred? How does this relate to our study of transformations earlier this semester?

36. Now let's suppose that Freddie Jr. is **y = F(–x)**
Complete the table.



y = F(–x)

x	–x	y
	–1	
	1	
	2	
	4	

37. On the coordinate plane above, graph the 4 ordered pairs (x, y). [*Hint: The 1st point should be (1, 1).*]

38. What type of transformation maps Fred, F(x), to Freddie Jr., F(–x)? (Be specific.)

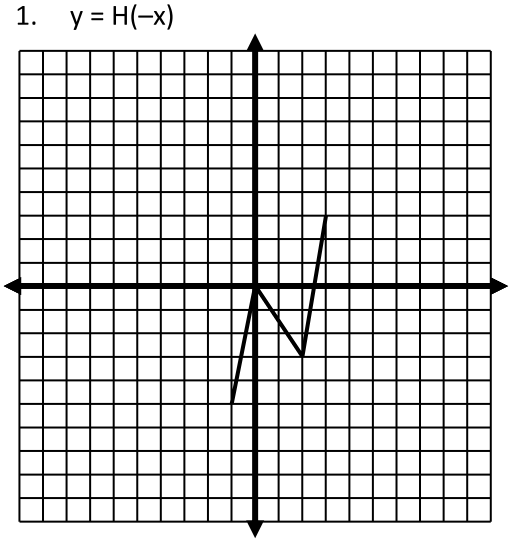
39. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

40. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

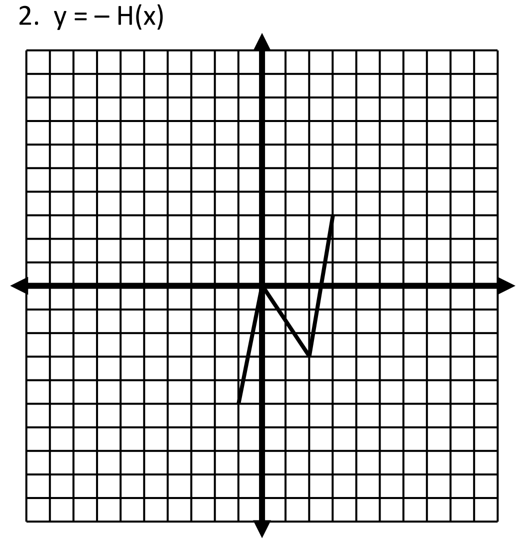
41. In y = F(–x), how did the negative coefficient of "x" affect the graph of Fred? How does this relate to our study of transformations earlier this semester?

****Checkpoint: Harry is H(x) and is shown on each grid. Use Harry's characteristic points to graph Harry's children without making a table.**

1. y = H(–x)



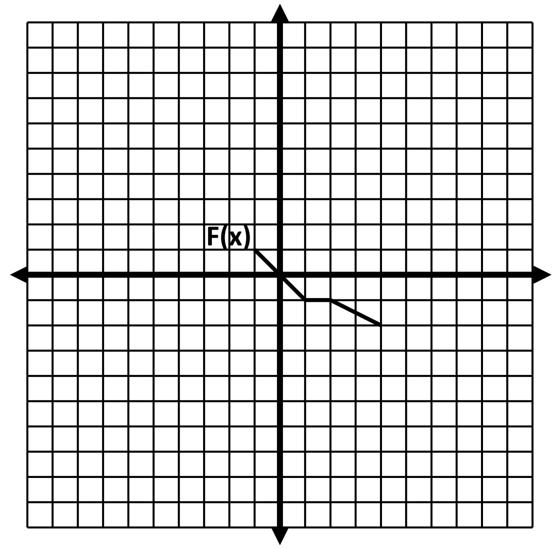
2. y = – H(x)



42. Now let's return to Fred, whose equation is **y = F(x)**.

Complete the chart with Fred's characteristic points.

x	**F(x)**



43. Let's suppose that Freddie Jr. is **y = 4 F(x)**
Complete the table.

y = 4 F(x)

x	**F(x)**	**y**
–1		
1		
2		
4		

44. On the coordinate plane above, graph the 4 ordered pairs (x, y). [*Hint: The 1st one should be (–1, 4).*]

45. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

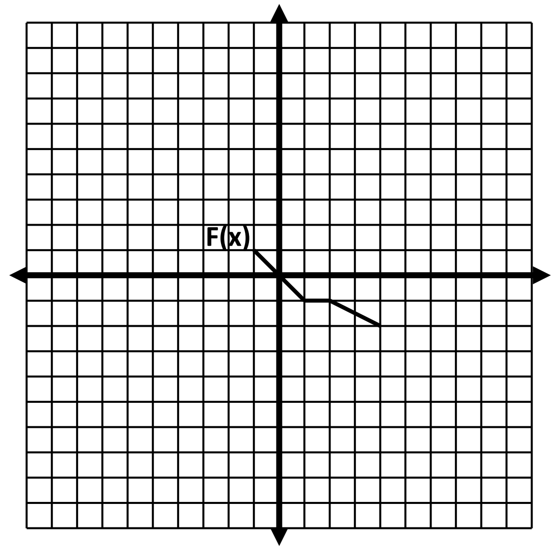
46. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

47. In y = 4 F(x), the coefficient of "F(x)" is 4. How did that affect the graph of Fred? Is this one of the transformations we studied? If so, which one? If not, explain.

48. Now let's suppose that Freddie Jr. is **y = ½ F(x)**.
Complete the table.

y = ½ F(x)

x	F(x)	y
–1		
1		
2		
4		



49. On the coordinate plane above, graph the 4 ordered pairs (x, y). [*Hint: The 1st one should be (–1, ½).*]

50. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

51. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

52. In y = ½ F(x), the coefficient of "F(x)" is ½. How did that affect the graph of Fred? How is this different from the graph of y = 4 F(x) on the previous page?

****Checkpoint:**

1. Complete each chart below. Each chart starts with the characteristic points of Fred.

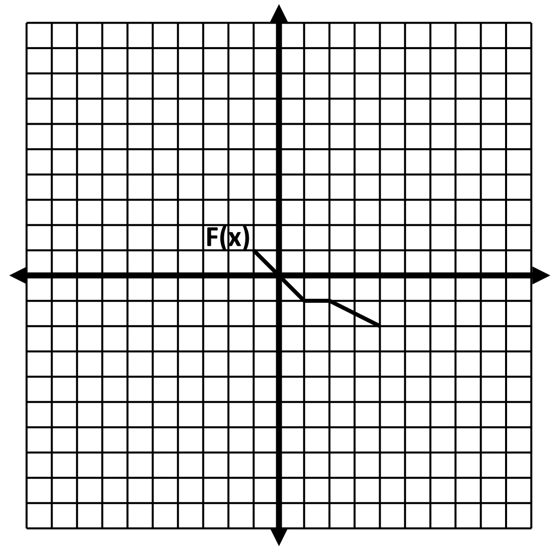
x	F(x)	3 F(x)
–1	1	
1	–1	
2	–1	
4	–2	

x	F(x)	¼ F(x)
–1	1	
1	–1	
2	–1	
4	–2	

2. Compare the 2nd and 3rd columns of each chart above. The 2nd column is the y-value for Fred. Can you make a conjecture about how a coefficient changes the parent graph?

53. Now let's suppose that Freddie Jr. is **y = –3 F(x)**.

Complete the table.



y = –3 F(x)

x	F(x)	y
–1		
1		
2		
4		

54. On the coordinate plane above, graph the 4 ordered pairs (x, y). [*Hint: The 1st one should be (–1, –3).*]

55. Reread the conjecture you made in #7 on the previous page. Does it hold true or do you need to refine it? If it does need some work, restate it more correctly here.

****Checkpoint: Let's revisit Harry, H(x).**

1. Describe the effect on Harry's graph for each of the following.

   Example: –5H(x) <u>Each point is reflected in the x-axis and is 5 times as far from the x-axis.</u>

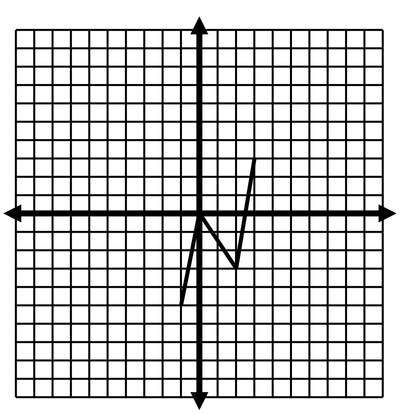
   a. 3H(x) ____

   b. –2H(x) ____

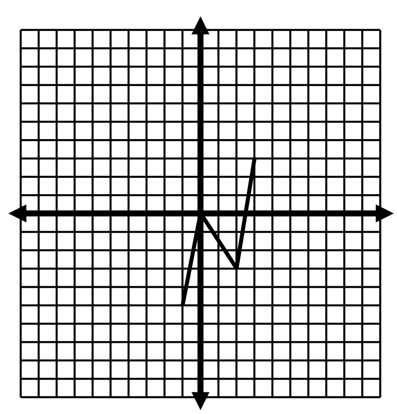
   c. $\frac{1}{2}$H(x) ____

2. Use your answers to questions 1 and 2 to help you sketch each graph *without using a table*.

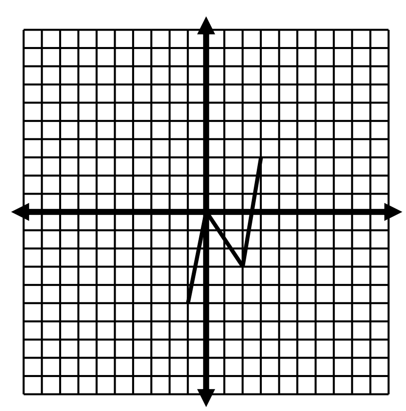
a. y = 3H(x)



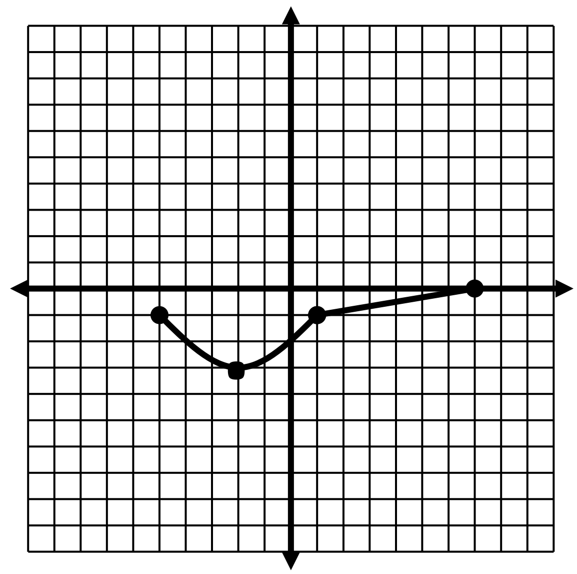
b. y = –2H(x)



c. y = $\frac{1}{2}$H(x)



56. The graph of **Dipper, D(x),** is shown.



List the characteristic points of Dipper.

____

What is different about Dipper from the functions we have used so far?

Since Dipper is our original function, we will refer to him as the **parent function**. Using our knowledge of transformational functions, let's practice finding children of this parent.

Note: In transformational graphing where there are multiple steps, it is important to perform the translations last.

I. **Example:** Let's explore the steps to graph **Dipper Jr, 2D(x + 3) + 5**, without using tables.

Step 1. The transformations represented in this new function are listed below in the order they will be performed. (See note above.)

- Vertical stretch by 2 (Each point moves twice as far from the x-axis.)
- Translate left 3.
- Translate up 5.

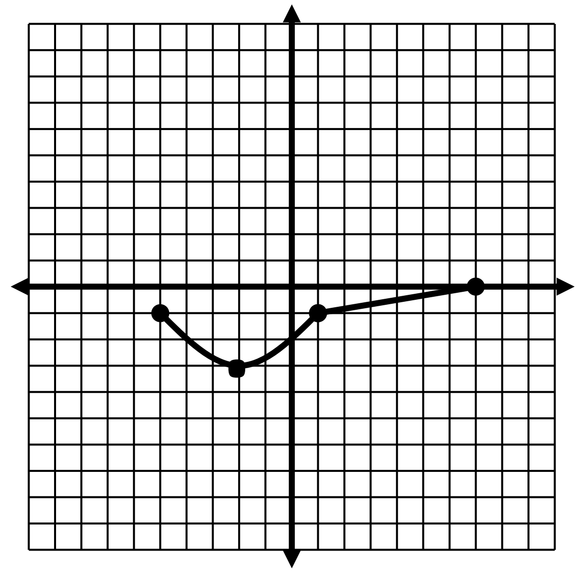
Step 2. On the graph, put your pencil on the left-most characteristic point, (– 5, –1) .

- Vertical stretch by 2 takes it to (– 5, –2). (Note that the originally, the point was 1 unit away from the x-axis. Now, the new point is 2 units away from the x-axis.)
- Starting with your pencil at (– 5, –2), translate this point 3 units to the left. Your pencil should now be on (– 8, –2).
- Starting with your pencil at (– 8, –2), translate this point up 5 units. Your pencil should now be on (– 8, 3).
- Plot the point (– 8, 3). It is recommended that you do this using a different colored pencil.

Step 3. Follow the process used in Step 2 above to perform all the transformations on the other 3 characteristic points.

Step 4. After completing Step 3, you will have all four characteristic points for Dipper Jr. Use these to complete the graph of Dipper Jr. Be sure you use a curve in the appropriate place. Dipper is not made of segments only.

II. Dipper has another child named **Little Dip, – D(x) – 4**



Using the process in the previous example as a guide, graph Little Dip (without using tables).

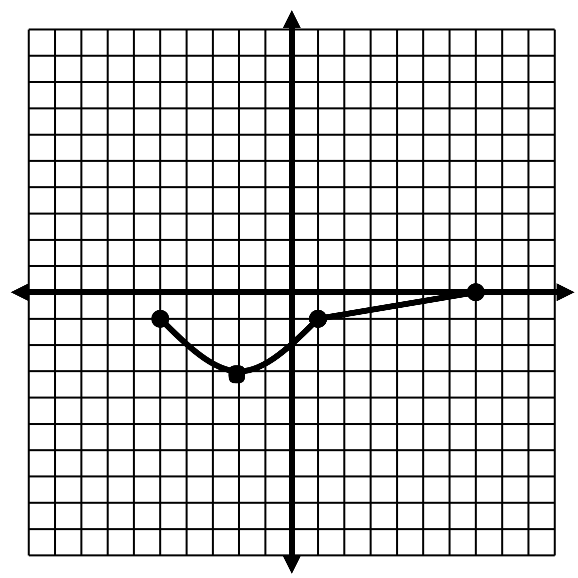
1. List the transformations needed to graph Little Dip.
   (Remember, to be careful with order.)

   - ____
   - ____

2. Apply the transformations listed above to each of the four characteristic points.

3. Complete the graph of Little Dip using your new characteristic points from #2.

III. Dipper has another child named **Dipsy, 3 D(– x)**



Using the process in the previous example as a guide, graph Dipsy (without using tables).

1. List the transformations needed to graph Dipsy.
   (Remember, to be careful with order.)

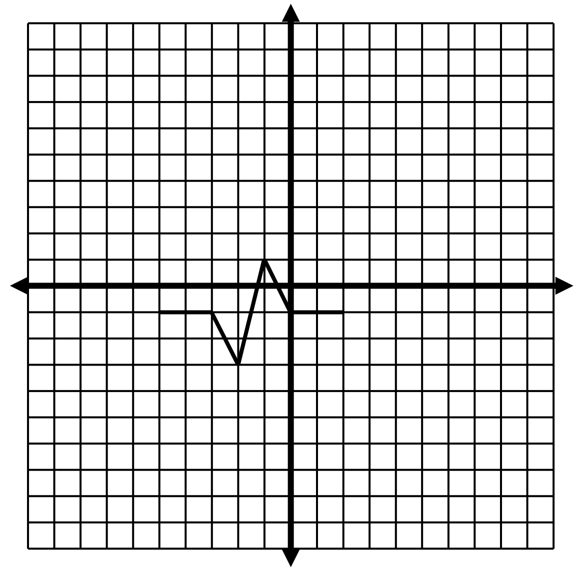
   - ____
   - ____

2. Apply the transformations listed above to each of the four characteristic points.

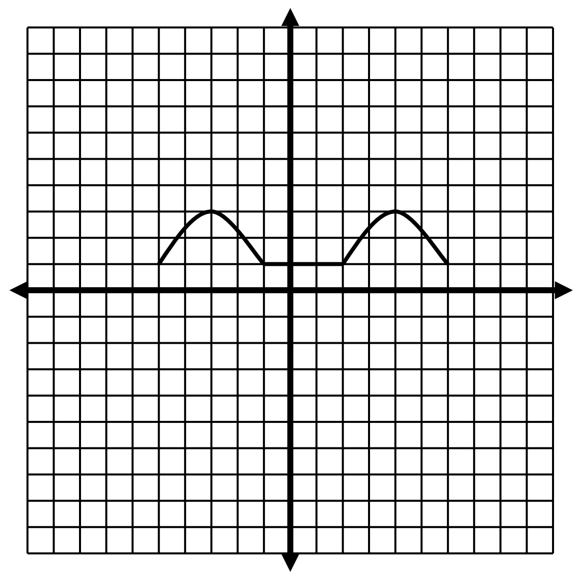
3. Complete the graph of Dipsy using your new characteristic points from #2.

IV. Now that we have practiced transformational graphing with Dipper and his children, you and your partner should use the process learned from the previous three problems to complete the following.

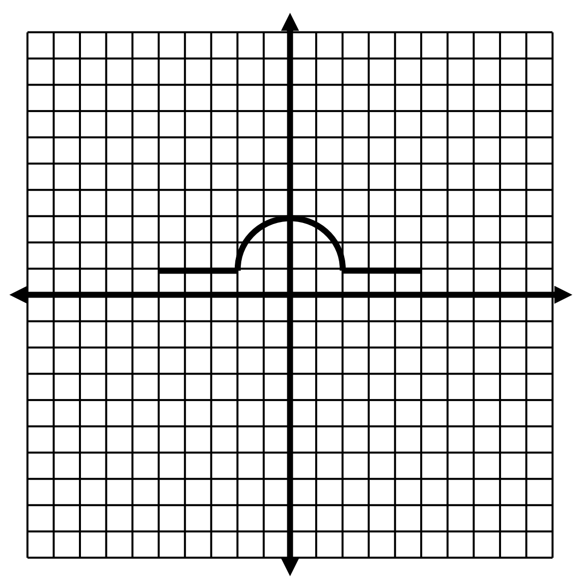
1. Given Cardio, C(x), graph: **y = 3C(x) + 5**



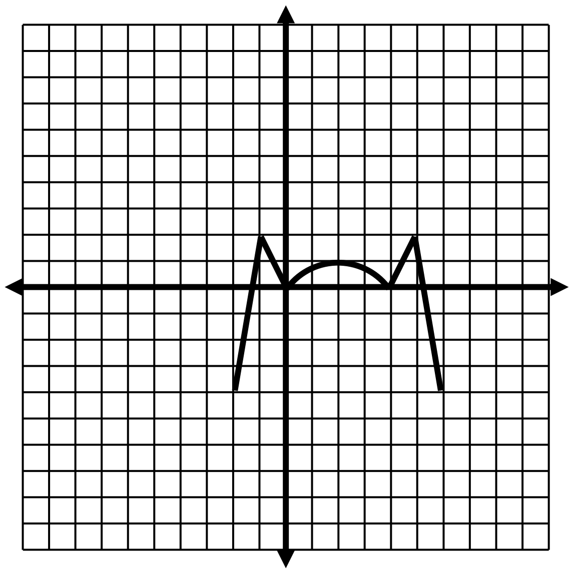
2. Given Garfield, G(x), graph: **y = – G(x – 3) – 6**



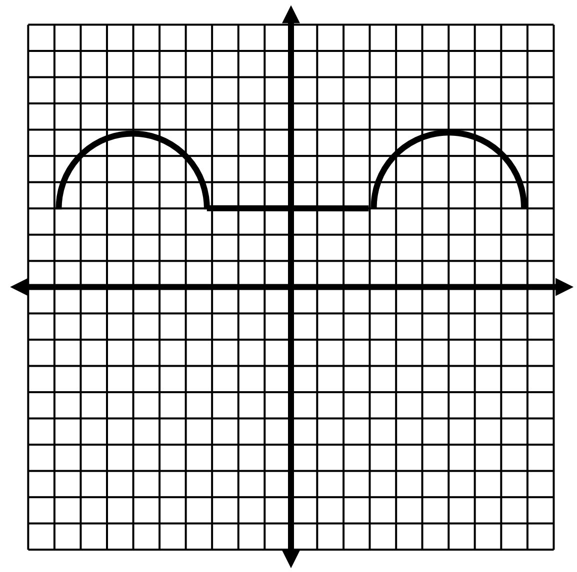
3. Given Horizon, H(x), graph **y = – 3H(x)**



4. Given Batman, B(x), graph: **$y = B(-x) + 8$**



5. Given Mickey, M(x), graph: **$y = -\frac{1}{3}M(x)$**

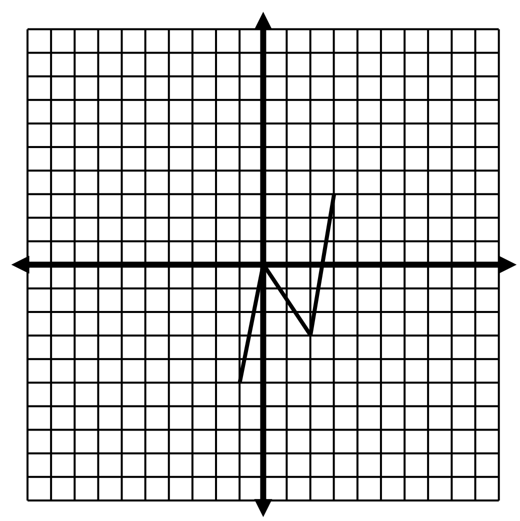


V. Finally, let's examine a reflection of Harry in the line y = x.

1. Graph this line (y = x) on the grid.

2. Using Harry's characteristic points and the MIRA, graph Harry's reflection.

3. Complete the charts below with the characteristic points:



Harry, y = H(x)

x	y

Harry's reflection in y = x:

x	y

4. Compare the points in the two charts. Describe what happens when we reflect in the line y = x.

(This should match what we learned in our earlier study of reflections in the line y = x.)

5. A reflection in the line y = x, shows a graph's **inverse**. We will study this in more depth in a future unit. Look at the graph of Harry's inverse. Is the inverse a function? Explain how you know.